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Combining post-Newtonian and Numerical Relativity results to describe coalescing compact binaries

Séminaire GReCO
IAP - 09/03/15

Outline

- Introduction: analytical and numerical descriptions of the coalescence
- Hybrid PN/NR waveforms with higher modes (aligned spins)
- Phenomenological inspiral-merger-ringdown model for precessing binaries

Motivation: building accurate templates for gw detection

- Coalescing binaries of compact objects (black holes and/or neutron stars) are one of the most promising sources of GW that we hope to detect with the advanced versions the ground based detectors LIGO and Virgo and with the future space-based detector eLISA.

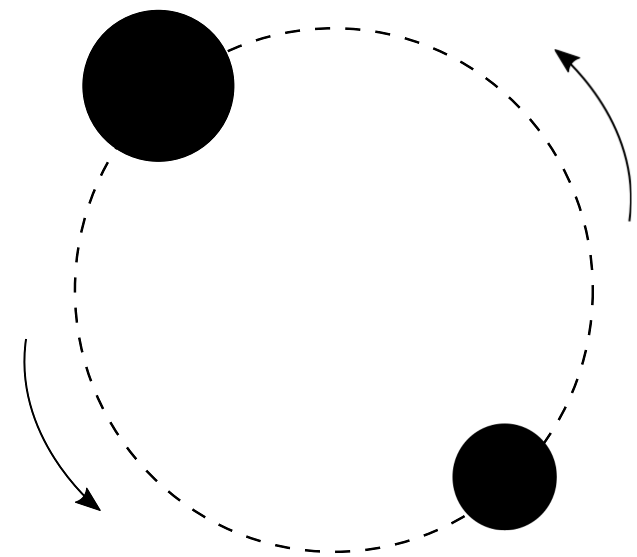
Neutron stars
$M \lesssim 3 M_\odot$

IMBH =?

Mass

Stellar Mass
Black Holes
$5 M_\odot \lesssim M \lesssim 10^2 M_\odot$

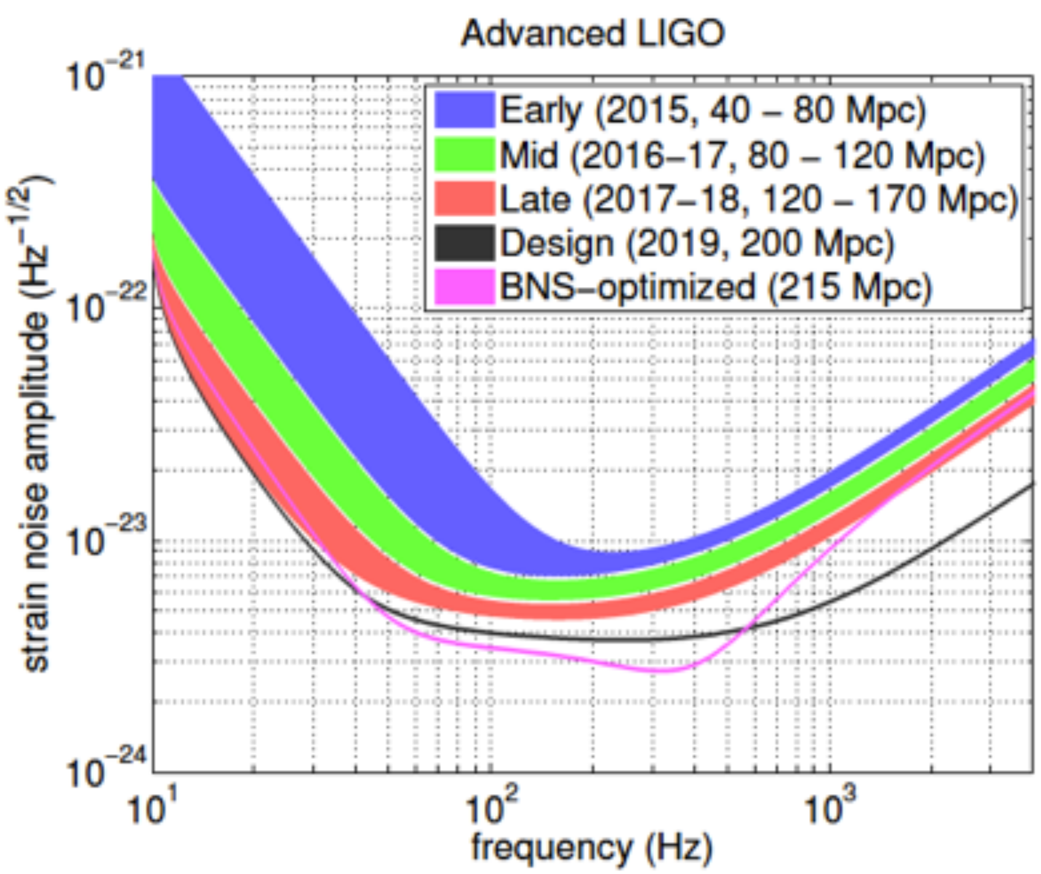
Super Massive
Black Holes
$10^6 M_\odot \lesssim M \lesssim 10^9 M_\odot$

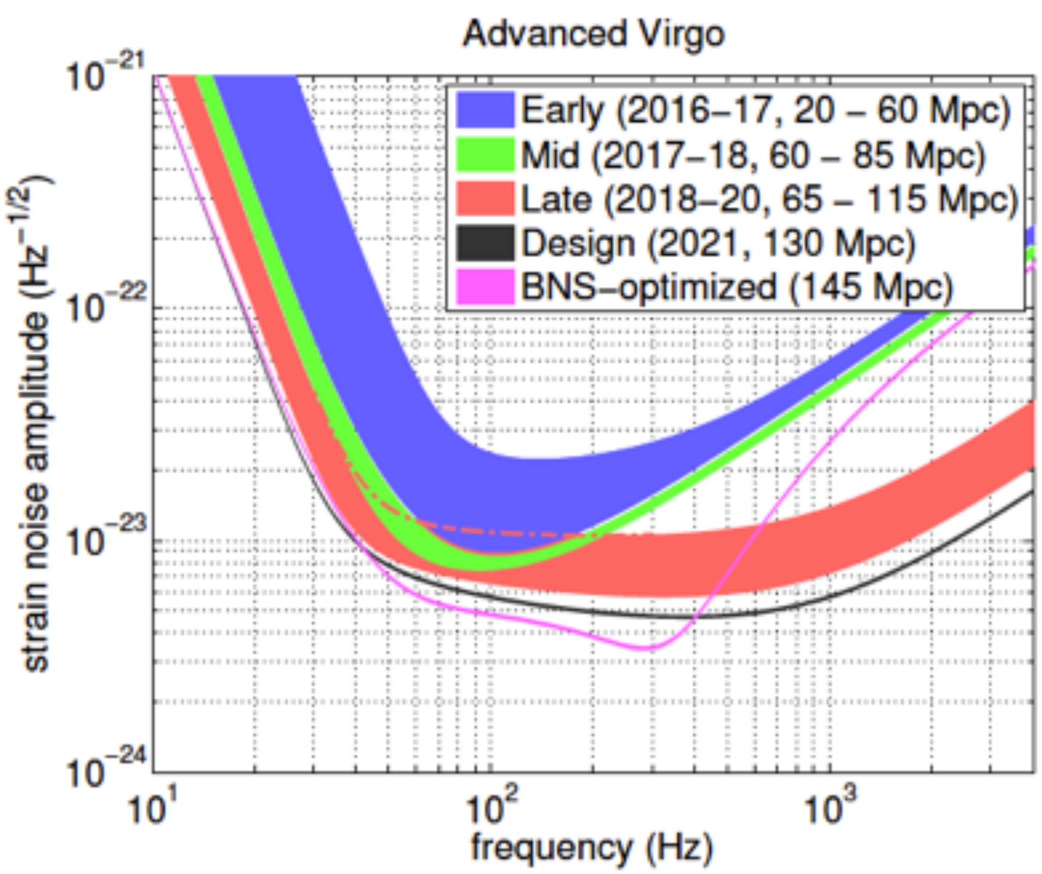


- Successfully extracting the very weak signal from the noise and estimating the parameters of the source with good precision can be achieved using matched filtering techniques provided that we have a very accurate modeling of the waveform.

Advanced Interferometer network

The advanced versions of the LIGO Virgo interferometers to start observing runs in 2015





LIGO/Virgo Collaboration
arXiv:1304.0670

Estimated rates

Epoch	Estimated Run Duration	$E_{GW} = 10^{-2} M_\odot c^2$ Burst Range (Mpc)		BNS Range (Mpc)		Number of BNS Detections	% BNS Localized within	
		LIGO	Virgo	LIGO	Virgo		5 deg^2	20 deg^2
2015	3 months	40 – 60	–	40 – 80	–	0.0004 – 3	–	–
2016–17	6 months	60 – 75	20 – 40	80 – 120	20 – 60	0.006 – 20	2	5 – 12
2017–18	9 months	75 – 90	40 – 50	120 – 170	60 – 85	0.04 – 100	1 – 2	10 – 12
2019+	(per year)	105	40 – 80	200	65 – 130	0.2 – 200	3 – 8	8 – 28
2022+ (India)	(per year)	105	80	200	130	0.4 – 400	17	48

LIGO/Virgo Collaboration
arXiv:1304.0670

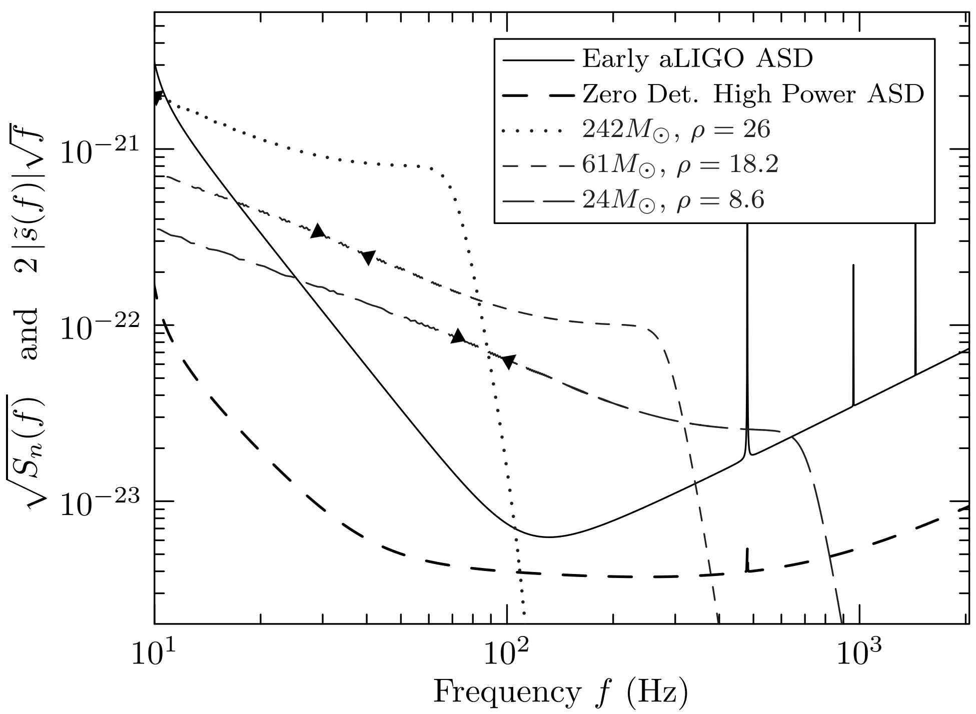
Table 5. Detection rates for compact binary coalescence sources.

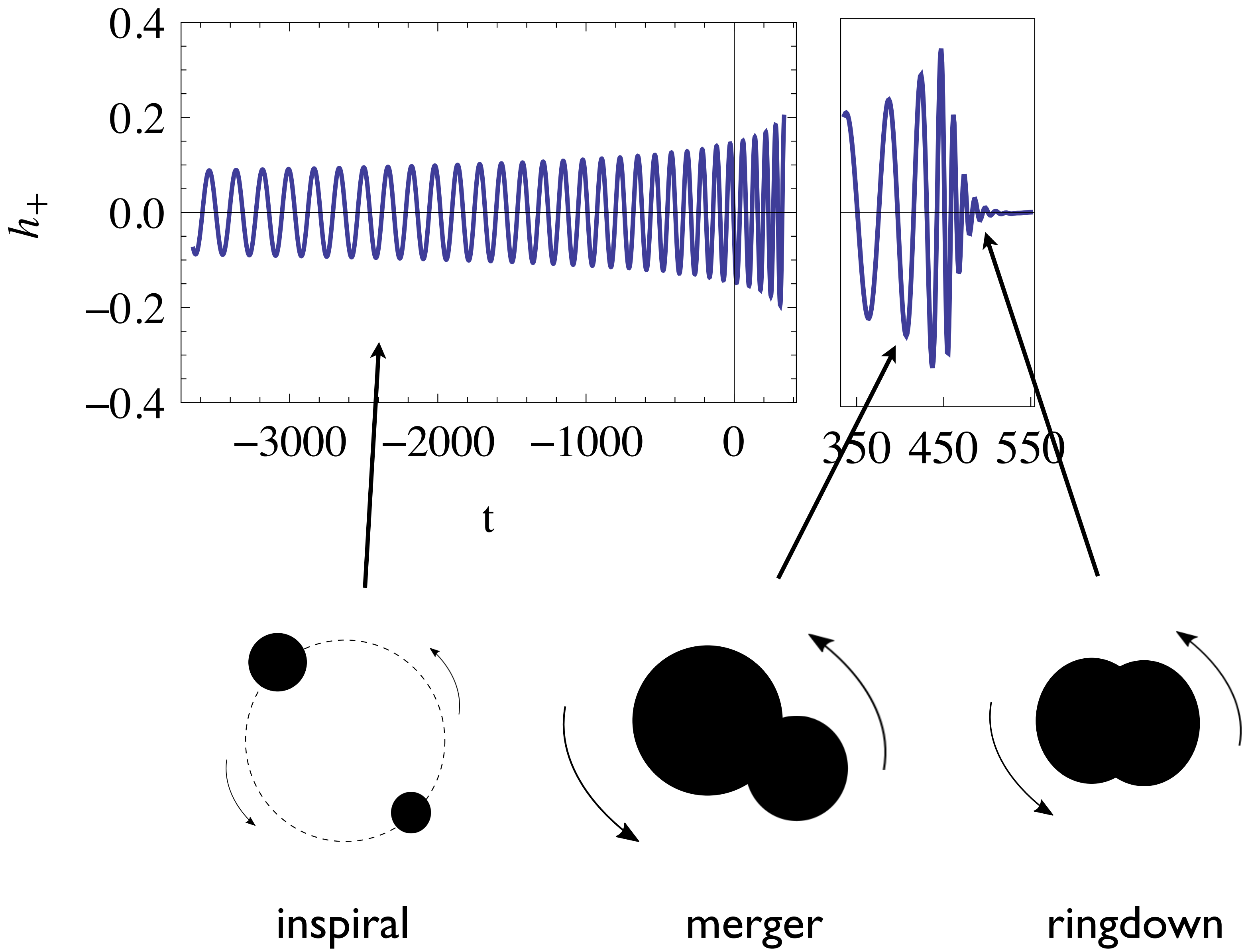
IFO	Source[a]	$\dot{N}_{low}$ yr^{-1}	$\dot{N}_{re}$ yr^{-1}	$\dot{N}_{high}$ yr^{-1}	$\dot{N}_{max}$ yr^{-1}
	NS–NS	2×10^{-4}	0.02	0.2	0.6
	NS–BH	7×10^{-5}	0.004	0.1	
Initial	BH–BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			<0.001[b]	0.01[c]
	IMBH-IMBH			10^{-4}[d]	10^{-3}[e]
	NS–NS	0.4	40	400	1000
	NS–BH	0.2	10	300	
Advanced	BH–BH	0.4	20	1000	
	IMRI into IMBH			10[b]	300[c]
	IMBH-IMBH			0.1[d]	1[e]

LIGO/Virgo Collaboration
Class.Quant.Grav. 27 (2010)

Dynamics of Compact Binary Coalescences

Loses energy by GW emission
→separation decreases
(and frequency increases)





To extract the signal from the instrumental noise (matched filtering), the waveform needs to be modeled with great accuracy

CBC: modeling the inspiral with PN

During the «slow» inspiral, while the objects are far from each other, a perturbative treatment is valid:

post-Newtonian expansion in v/c

Newtonian estimate

$$\frac{1}{2}\mu v^2 = \frac{1}{2}\frac{Gm\mu}{r} \quad \text{i.e.} \quad \frac{v^2}{c^2} = \frac{R_s}{2r} \qquad R_s = 2\frac{Gm}{c^2}$$

- Purely analytical approach: iterate Einstein equations in harmonic coordinates

rewrite Einstein eqs

$$h^{\mu\nu} = \sqrt{-g}g^{\mu\nu} - \eta^{\mu\nu} \longrightarrow$$

$$\partial_\mu h^{\alpha\mu} = 0 \quad \text{harmonic gauge}$$

$$\Box h^{\mu\nu} = \frac{16\pi G}{c^4}\tau^{\mu\nu}$$

$$\tau^{\mu\nu} = |g|T^{\mu\nu} + \frac{c^4}{16\pi G}\Lambda^{\mu\nu}$$

$\tau^{\mu\nu}$ stress-energy pseudo tensor of matter + gravitational fields

(also 2 different approaches ADM and EFT)

- The formalism is based on an elegant combination of post-Minkowskian, post-Newtonian et multipolar expansions (see Living Review by Blanchet)

- To make the calculation tractable: effective description in terms of (spinning) point particles (regularisation UV)

State of the art in PN

state of the art for the phase for quasi circular orbits:

- non-spinning: 3.5 PN
- spin-orbit: 4 PN Marsat, Bohe, Blanchet, Buonanno (13)
- (aligned) spin-spin: 3PN Bohe, Faye, Marsat, Porter
- cubic-in-spin: Marsat (14)

$$\frac{dE}{dt} = -\mathcal{F} \implies \frac{d\omega}{dt} = \frac{-\mathcal{F}}{dE/d\omega}$$

$$x = \left(\frac{Gm\omega}{c^3}\right)^{2/3} = \mathcal{O}((v/c)^2)$$

$$E = -\frac{\mu c^2 x}{2}\Big[1 + e_1 x + e_2\, x^2 + e_3\, x^3 + e_4\, x^4 + e_{1.5}^{SO} x^{3/2} + e_{2.5}^{SO} x^{5/2} + e_{3.5}^{SO} x^{7/2} + e_2^{SS} x^2 + e_3^{SS} x^3 + e_{3.5}^{SSS} x^{7/2}\Big] + \mathcal{O}(x^{9/2}, x^{4.5}, x^4)$$

$$\mathcal{F} = \frac{32c^5}{5G} x^5 \nu^2 \Big[1 + f_1 x + f_{1.5}\, x^{3/2} + f_2\, x^2 + f_{2.5}\, x^{5/2} + f_3\, x^3 + f_{3.5}\, x^{7/2} + f_{1.5}^{SO} x^{3/2} + f_{2.5}^{SO} x^{5/2} + f_3^{SO} x^3 + f_{3.5}^{SO} x^{7/2} + f_4^{SO} x^4 + f_2^{SS} x^2 + f_3^{SS} x^3 + f_{3.5}^{SSS} x^{7/2}\Big] + \mathcal{O}(4, 4.5, 3.5)$$

For the full polarizations:

Non spinning: (2,|2|), (3,|3|) and (3,|1|) modes to 3.5 PN
all other modes to 3PN
All spin effects known to 2PN

Blanchet's Living review (14)
Faye, Blanchet, Marsat, Iyer (12)
Faye, Blanchet, Iyer (14)

Arun, Buonanno, Faye, Ochsner (09)
Buonanno, Faye, Hinderer (13)

NR simulations for the Merger

Non linearities become too strong: PN expansion breaks down
→ need to resort to Numerical Relativity
simulation of the full Einstein equations in vacuum

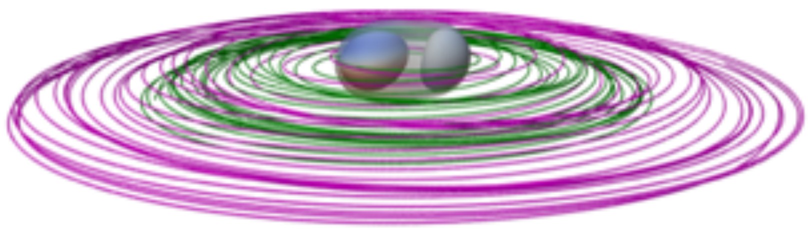


Image from Scheel et al. (14)

Very expensive: O(100) configs. only (a few 10^5 CPU hours/config)
public SXS catalog

Going to low frequencies is very expensive $\tau_{\rm coalescence} \approx \nu^{-1} f_{initial}^{-8/3}$ + instabilities + boundaries

Typically, simulations span O(10) orbits before the merger (see however Szilagyi et al. (2015))

Going to large mass ratios is very expensive $1 \leq q \leq 18$

very different scales to resolve
longer time to merger

Going to large spins is expensive $\chi \sim .994$ Scheel et al. (14)

Intrinsic parameter space is 7D: mass ratio + 6 spin components. Impossible to sample

For DA purposes, we need analytical models calibrated to simulations

Progress

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  Calderón et al., arXiv:1501.00918, submitted to PRD

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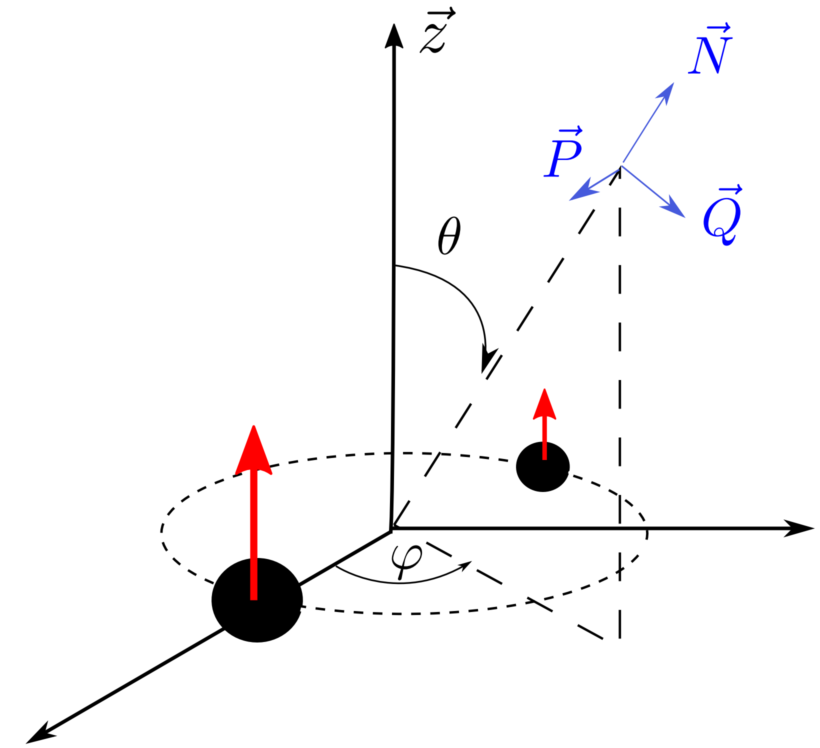
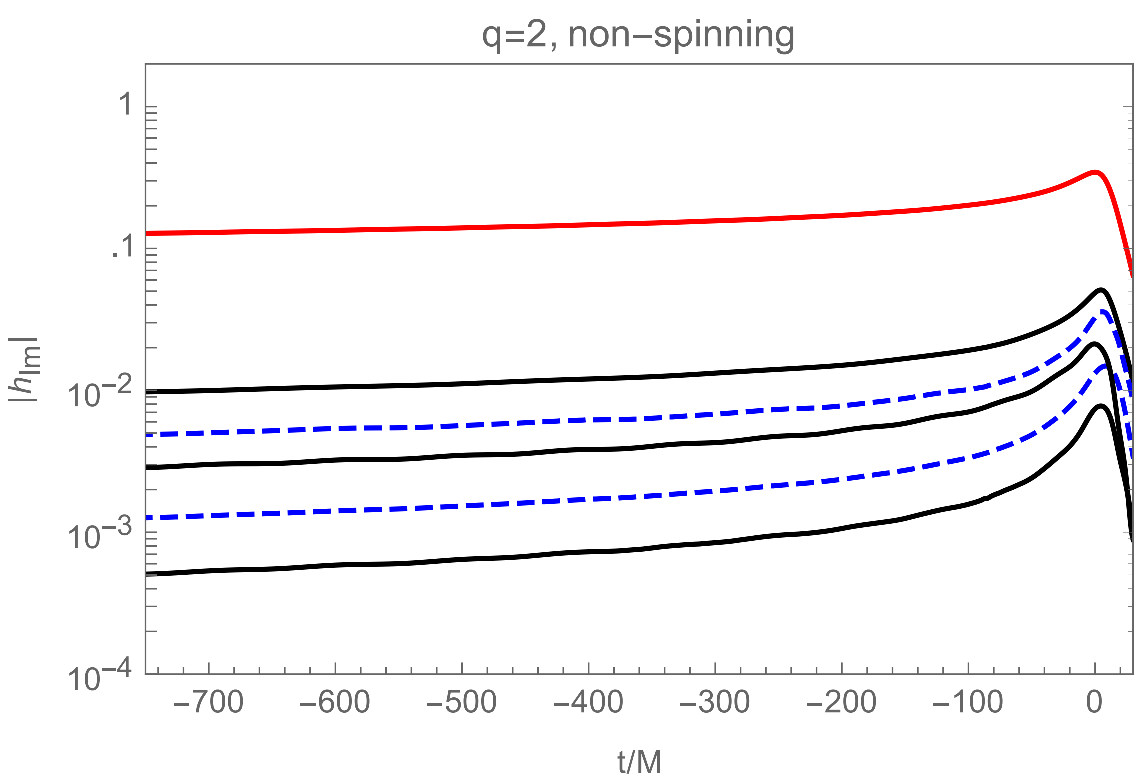
Waveform Modes

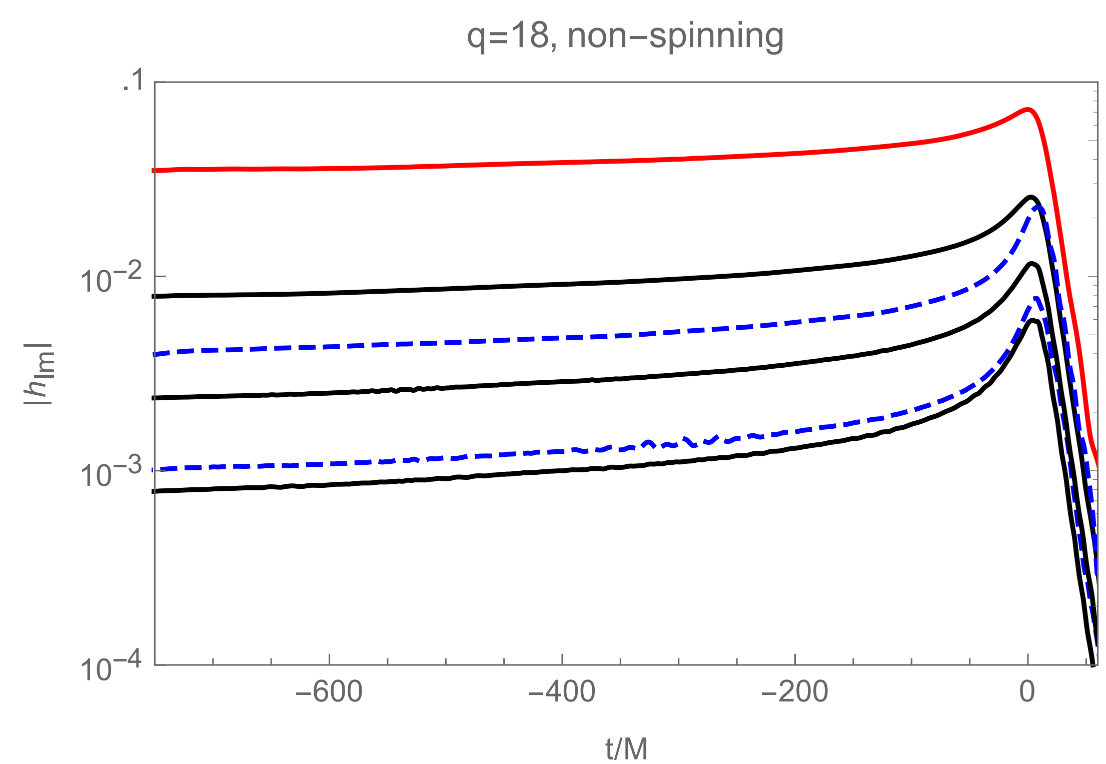
$$h(t,\theta,\varphi;\Xi) = h_+(t,\theta,\varphi;\Xi) - ih_\times(t,\theta,\varphi;\Xi)$$

$$h(t,\theta,\varphi;\Xi) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} Y^{-2}_{\ell,m}(\theta,\varphi)h_{\ell m}(t,\Xi)$$

Symmetry for aligned spins: $h_{\ell m}(t,\Xi) = (-1)^\ell h^*_{\ell,-m}(t,\Xi)$

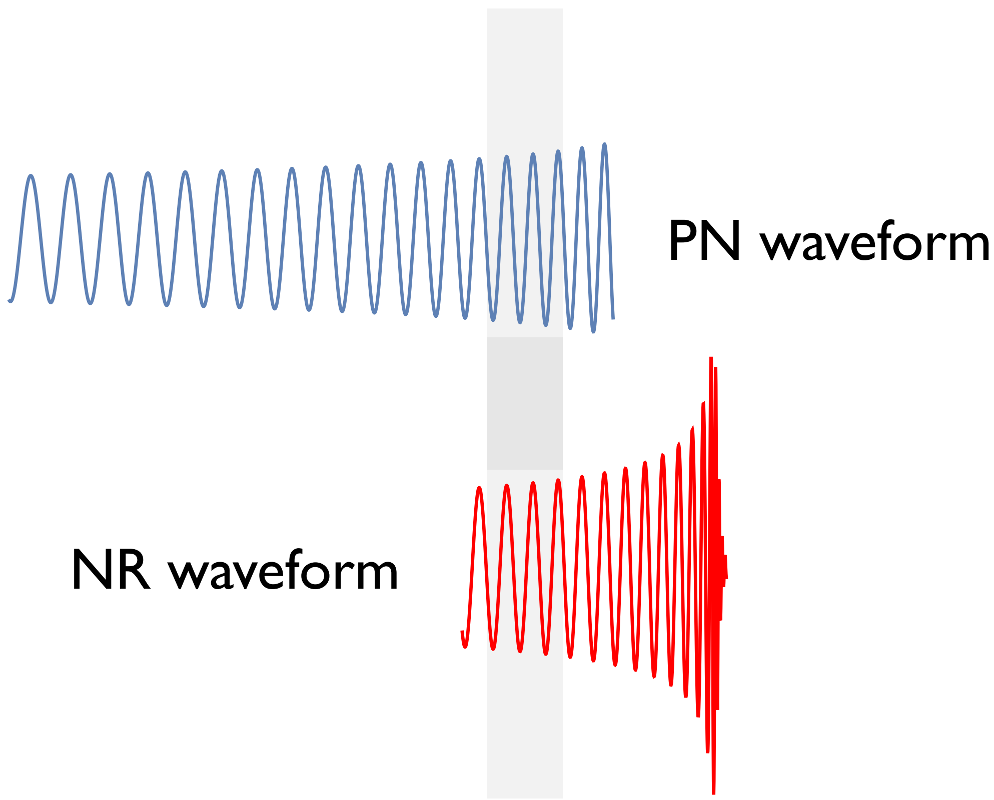






Higher modes suppressed by powers of v/c and mass asymmetry

Hybrids and Waveform alignement



Hybrid waveform produced by stitching together two aligned waveforms over some suitable window

$$h(t,\theta,\varphi;\Xi) = h_+(t,\theta,\varphi;\Xi) - i h_\times(t,\theta,\varphi;\Xi)$$

Convention freedom:

- time shift
- def of azimuthal angle
- def of polarization

Ideally, if both waveforms were infinitely accurate, they would satisfy

$$h^A(t,\theta,\varphi) = e^{i\psi_0} h^B(t+\tau,\theta,\varphi+\varphi_0)$$

or equivalently, their modes would satisfy $h^A_{\ell m}(t) = e^{i(\psi_0 + m\varphi_0)} h^B_{\ell m}(t+\tau)$

with $\psi_0 \in \{0,\pi\}$ to preserve the symmetry property $h_{\ell m}(t,\Xi) = (-1)^\ell h^*_{\ell,-m}(t,\Xi)$

Aligning consists in determining the best (τ,ϕ_0,ψ_0) from the waveforms.

Hybrid waveforms: (2,2) mode

$$h^A_{\ell m}(t) = e^{i(\psi_0 + m\varphi_0)} h^B_{\ell m}(t+\tau)$$

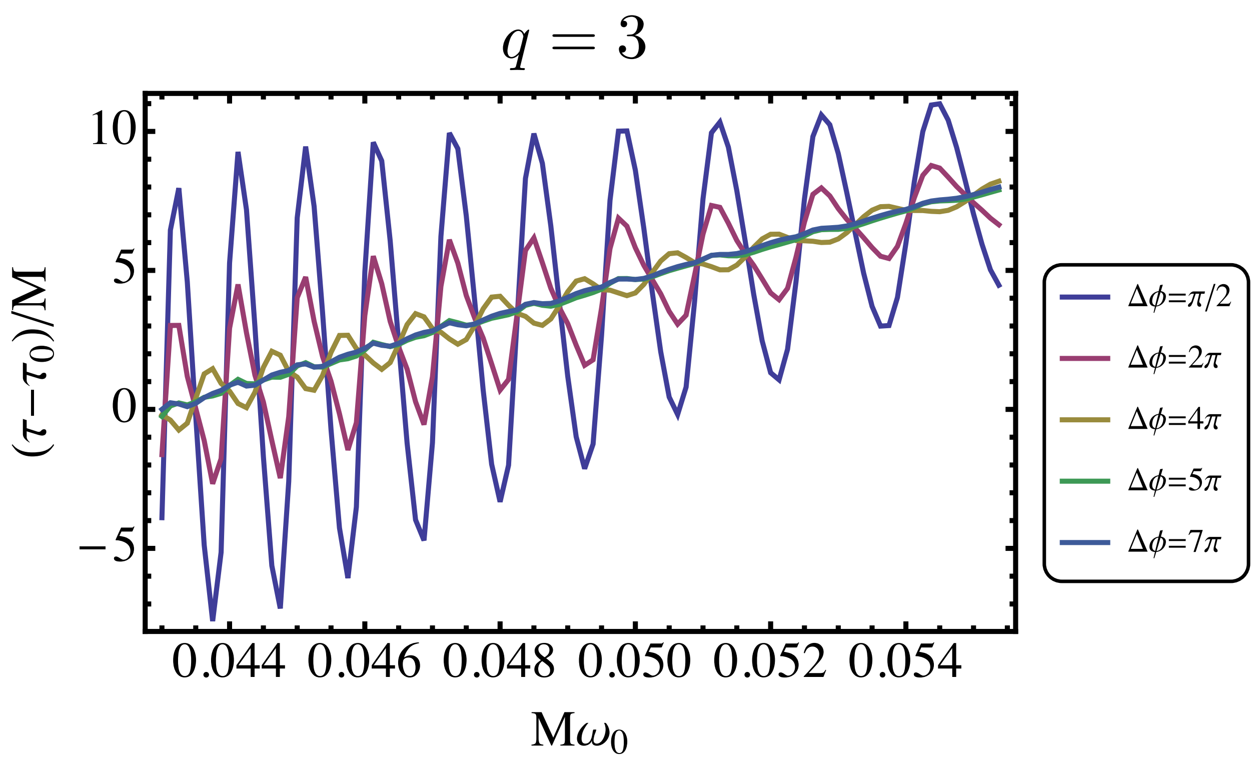
How to choose a suitable window:

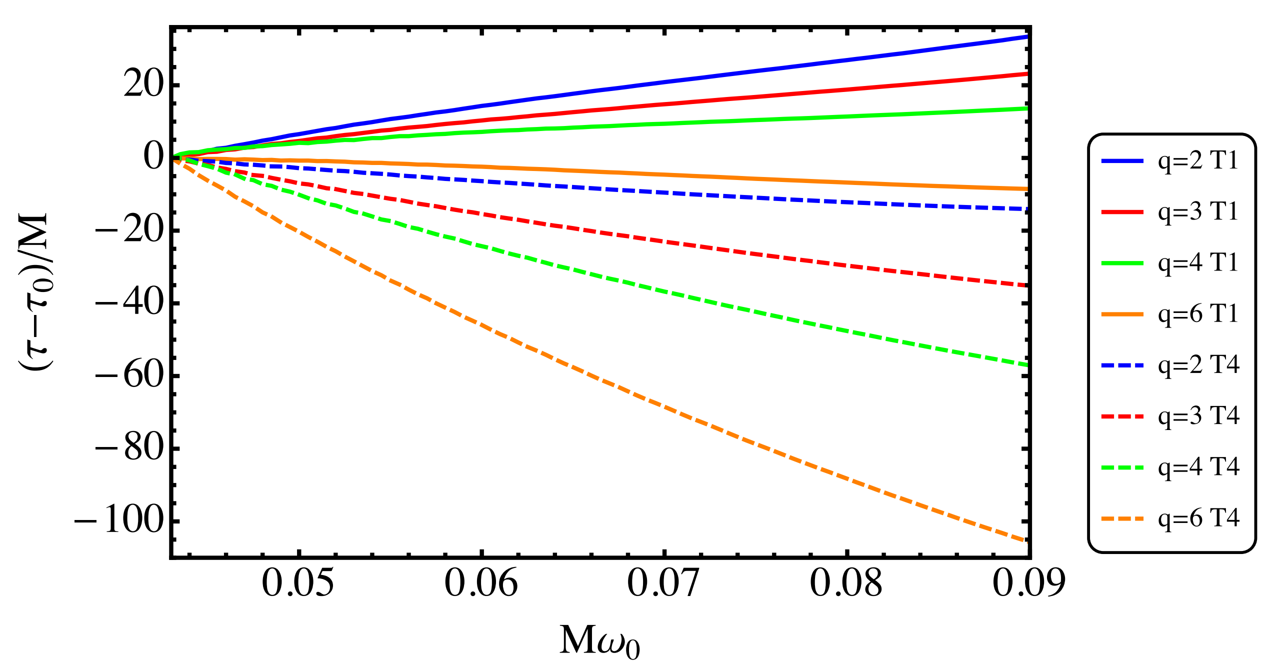
- as early as possible (PN loses accuracy)
- late enough to avoid junk radiation
- long enough to remove NR oscillations (eccentricity…)
  (just as a reference, Schwarzschild ISCO ~.14)

1- Determine timeshift by comparing the frequency over the window (other choices possible)

$$\Delta(\tau; t_0, \Delta t) = \int_{t_0}^{t_0+\Delta t} \left(\omega^{PN}(t) - \omega^{NR}(t-\tau)\right)^2 dt$$

2 - Just align the phases e.g. at the center of the window





Hybrid waveforms with higher modes (alignement)

Now 3 parameters (τ, ϕ_0, ψ_0) and one obviously cannot hybridize mode per mode independently.

How to use the different modes to constrain these parameters?

- just hybridize the full waveform at a given sky position (very impractical)
- some amplitude weighted combination ? (subdominant modes noisier…)
- use the (2,2) mode as much as possible!

1- Determine timeshift by comparing the frequency of (2,2) over the window

$$\Delta(\tau; t_0, \Delta t) = \int_{t_0}^{t_0+\Delta t} \left(\omega_{2,2}^{PN}(t) - \omega_{2,2}^{NR}(t-\tau)\right)^2 dt$$

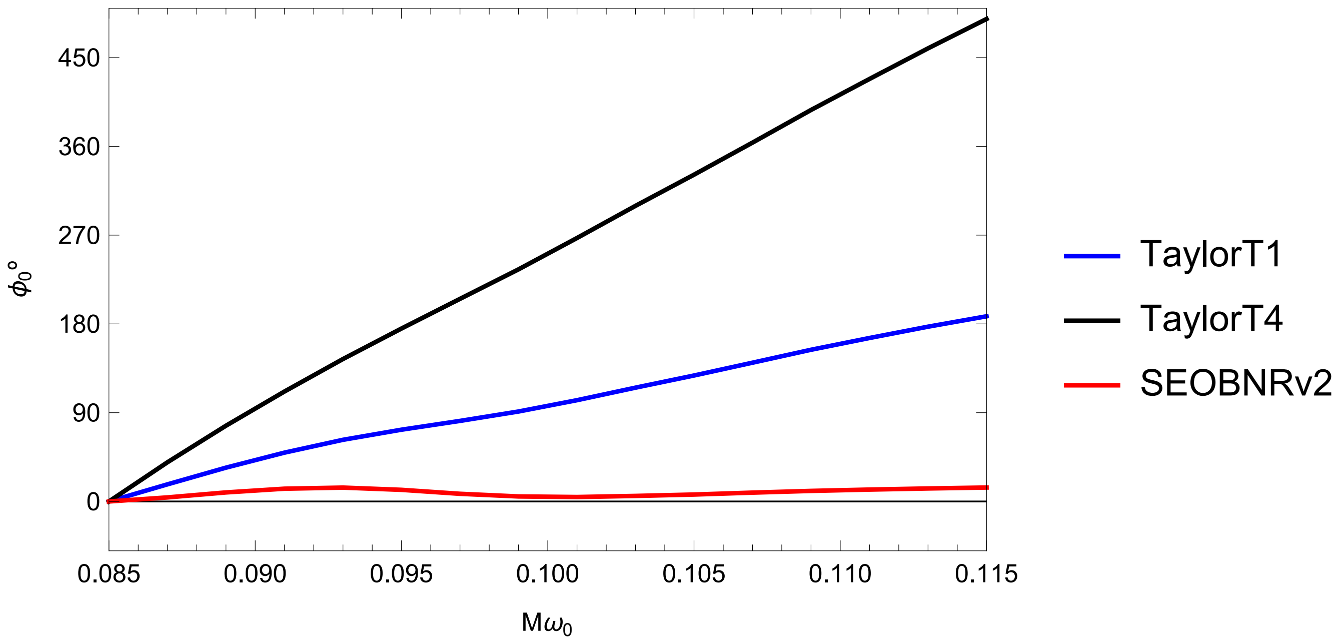
2 - Determine most of the 2 angular degrees of freedom using the (2,2) mode

$$(\psi_0, \varphi_0) = \left(\kappa\pi, -\frac{\Delta\phi_{2,2}}{2} + \left(\kappa' - \frac{\kappa}{2}\right)\pi \mod 2\pi\right)$$

$$\Delta\phi_{\ell m} = \phi_{\ell m}^{\rm NR}(t_0 - \tau) - \phi_{\ell m}^{\rm PN}(t_0)$$

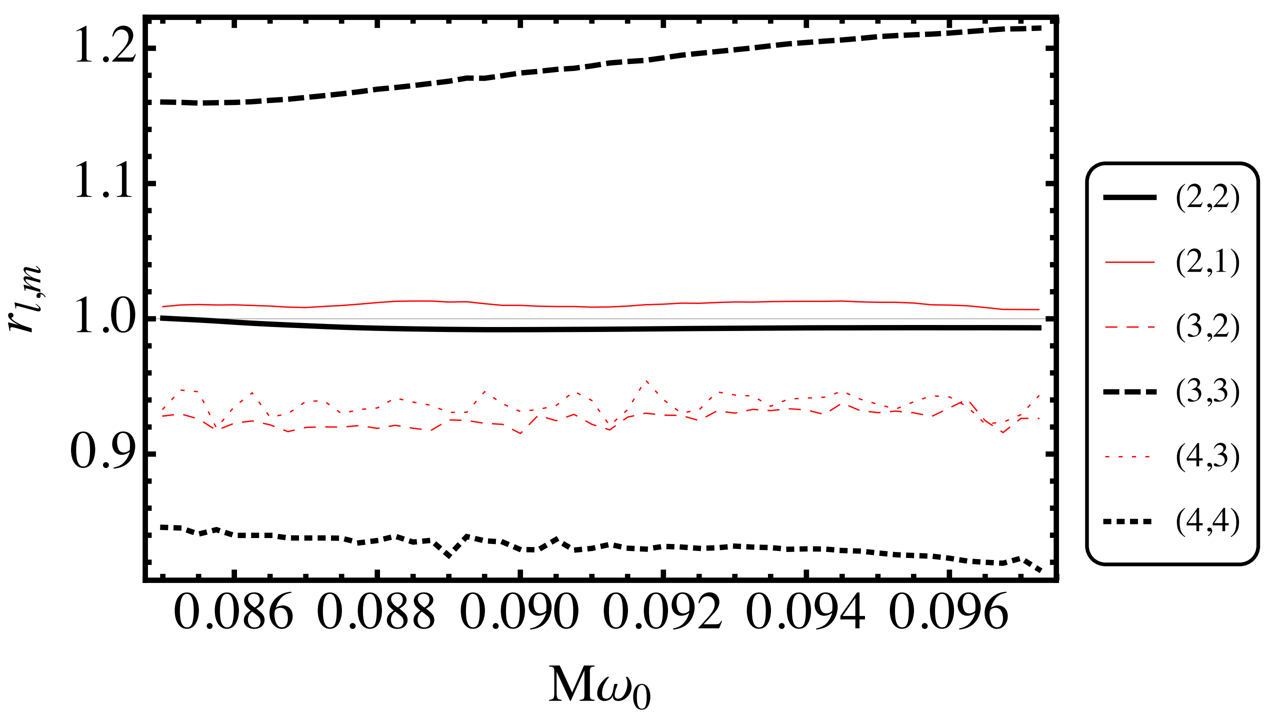
3 - Break the degeneracy using the second strongest mode (usually (3,3) mode, unless not present for symmetry reasons…)

Example: q=18, non spinning, TaylorT1

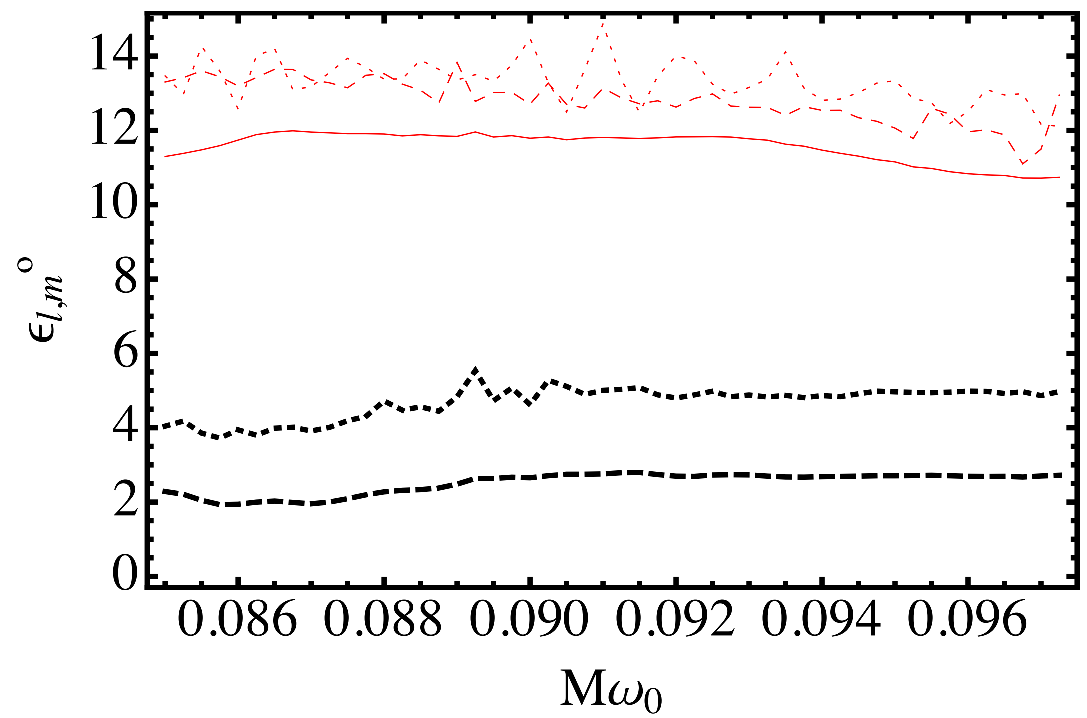


secular "orbital" dephasing just as in the (2,2) mode case

We want to quantify the additional residual errors due to the higher modes



Amplitude ratio at the center of the matching window



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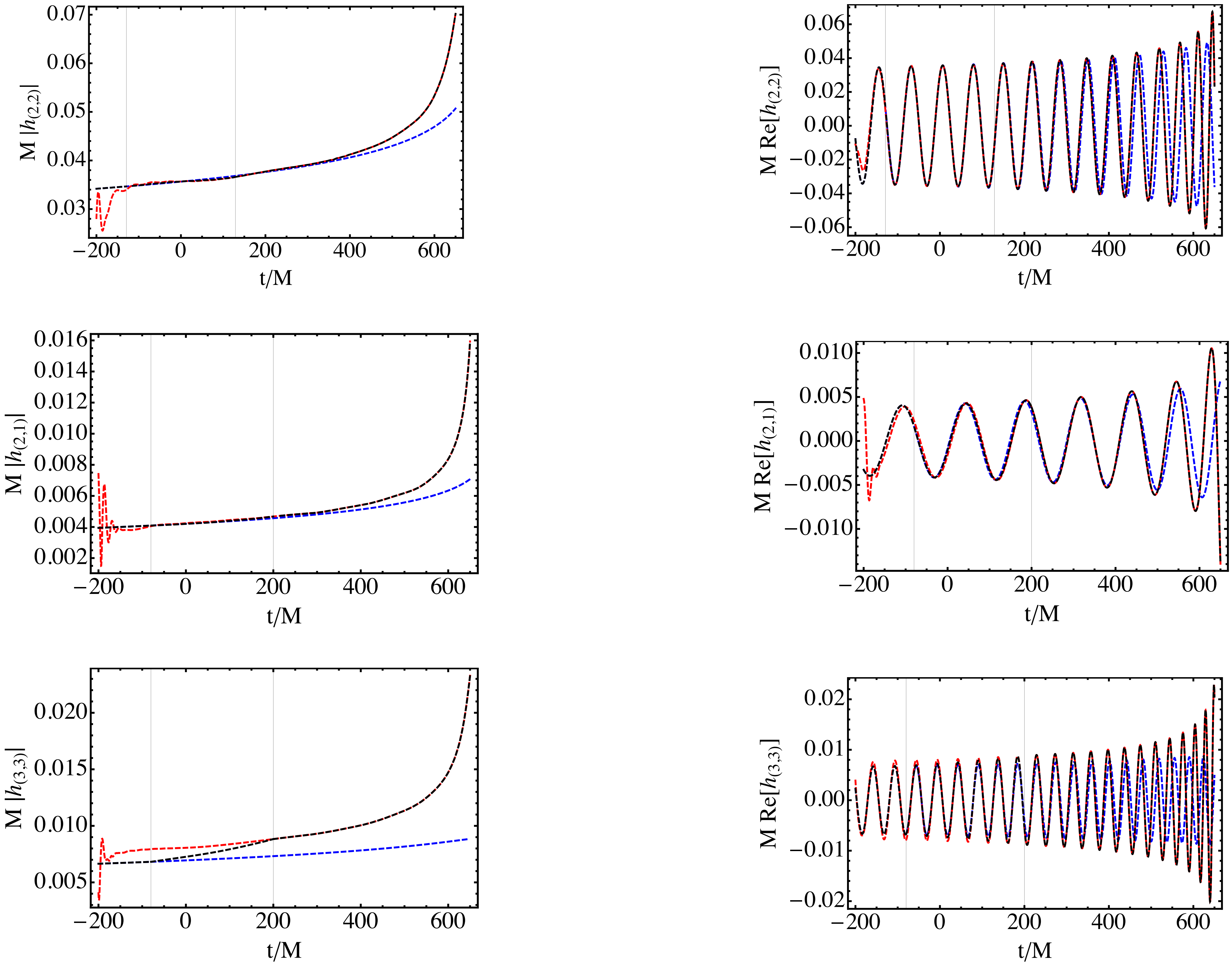
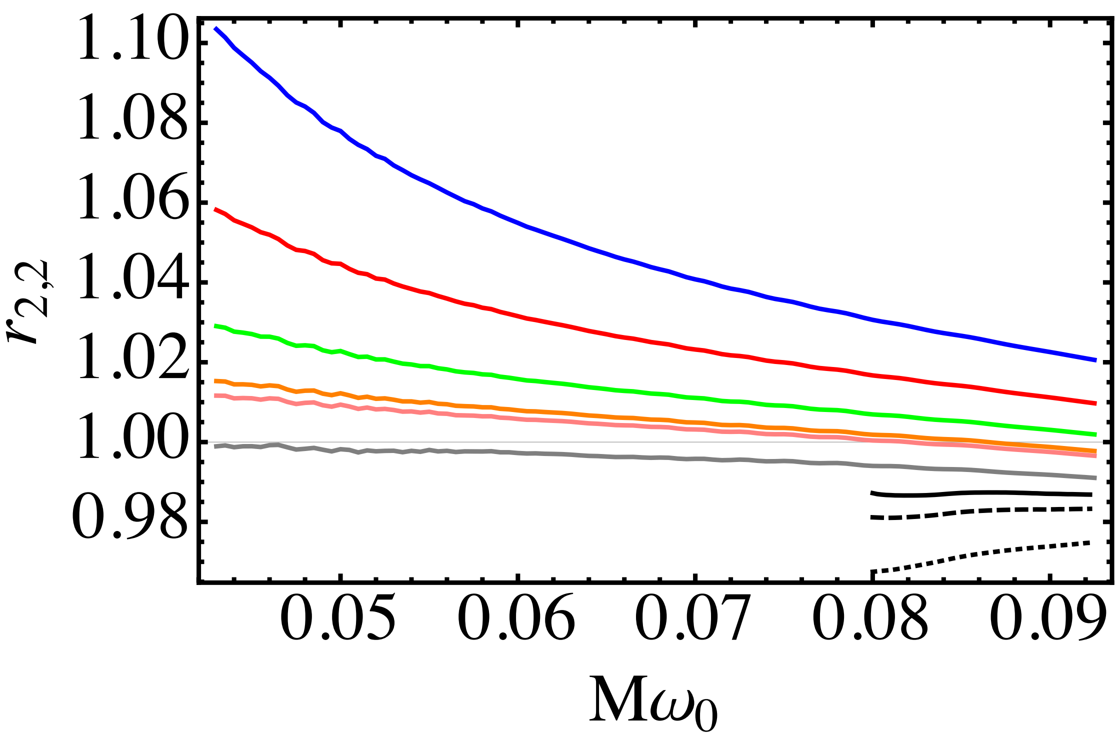


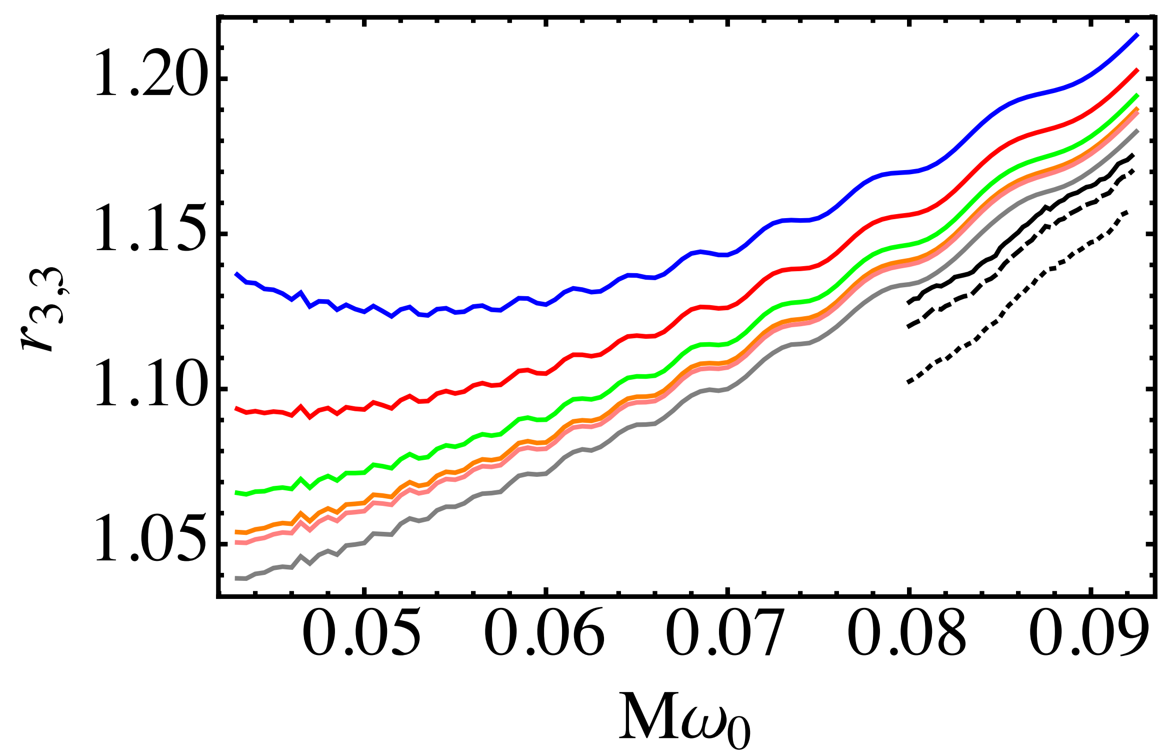
Origin of the amplitude discrepancies

Amplitude ratio at the center of the matching window $r_{\ell m} = \frac{|h^{\rm NR}_{\ell m}(t_0 - \tau)|}{|h^{\rm PN}_{\ell m}(t_0)|}$

For q=8 (non-spinning), we have waveforms from two different NR codes (BAM, SpEC).
PN approximant: TaylorT1. Amplitude corrections (2,2) mode to 3.5PN, (3,3) to 3PN.

Vary the extraction radius of the waves





Competing effects:

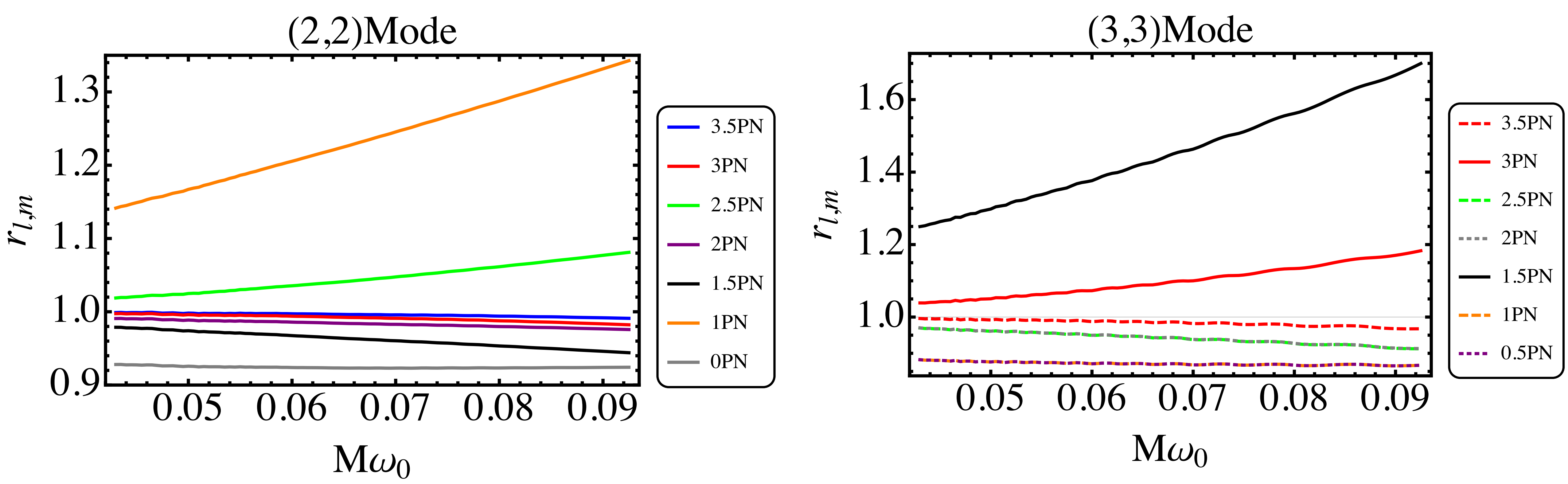
- PN more accurate at low frequencies
- NR extraction deeper in the wavezone at higher frequencies (gauge/code dependent)

Origin of the amplitude discrepancies

For some modes like (2,2), (2,1), the agreement is to the 1% level for large enough extraction radii (or extrapolated)

For other modes such as (3,3), (4,4)… larger disagreement even for extrapolated waves. The error is dominated by PN truncation error.

SpEC extrapolated vs TaylorT1 varying the PN order of the amplitude corrections:



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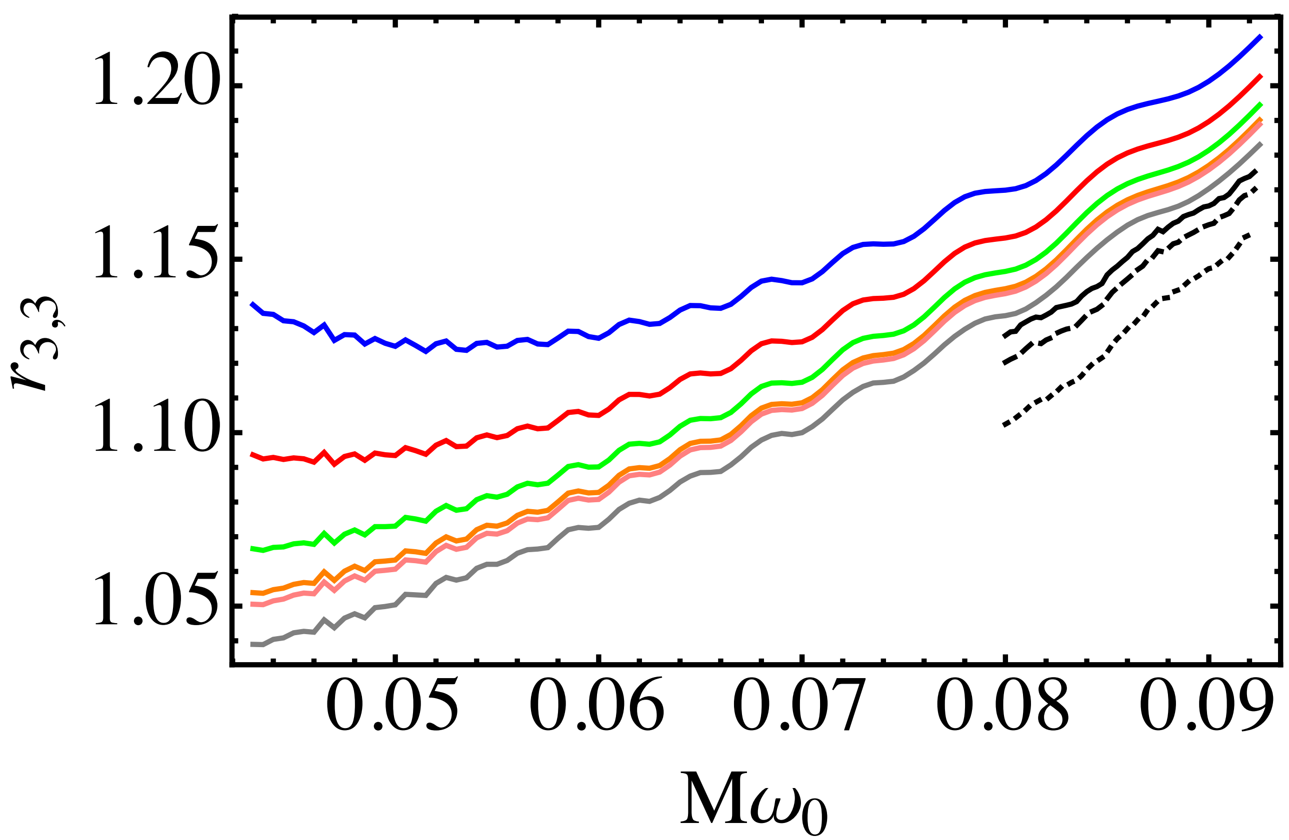
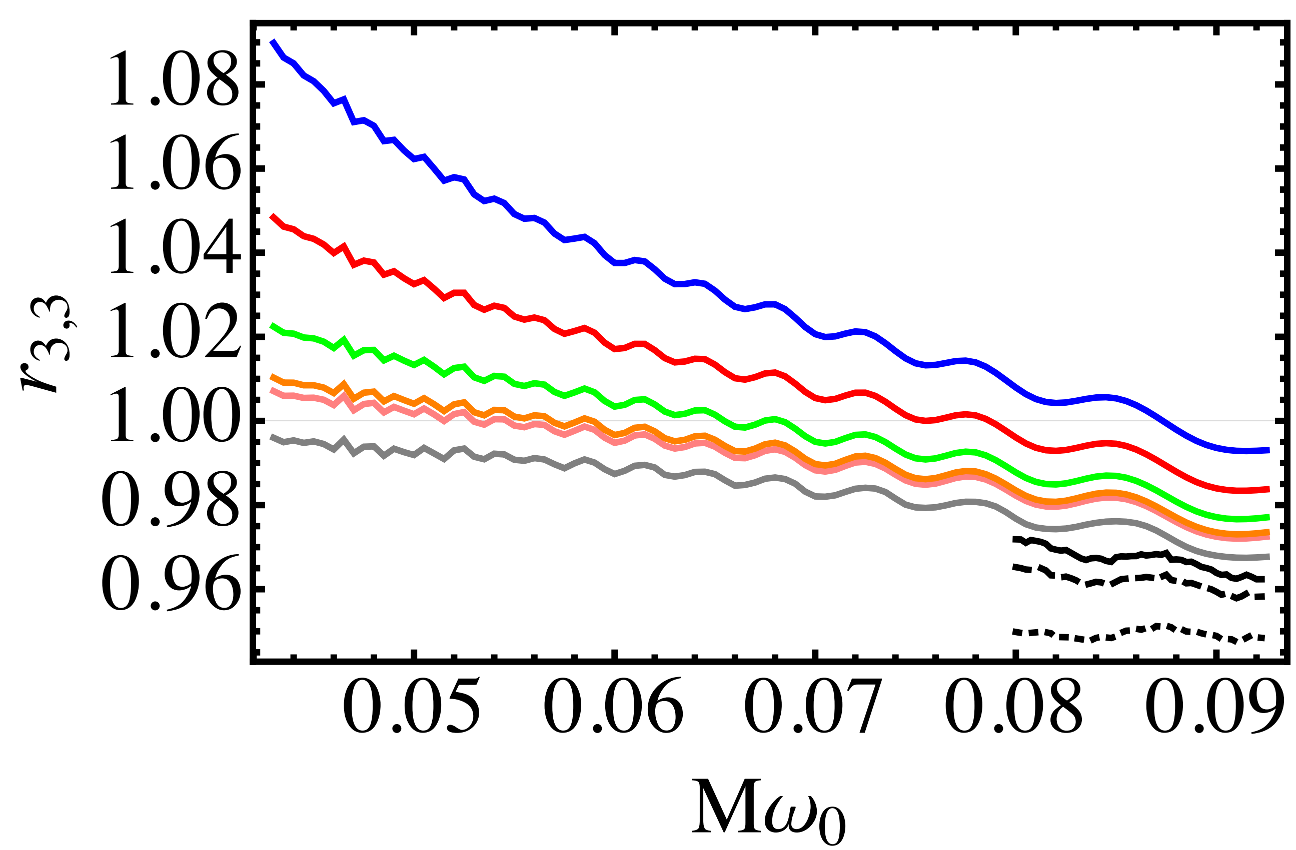


Adding the 3.5PN correction to the (3,3) mode

Faye, Blanchet, Iyer (2015)



Brings the disagreement down to ~2% on the amplitude of the first subdominant mode

Origin of the phase discrepancies

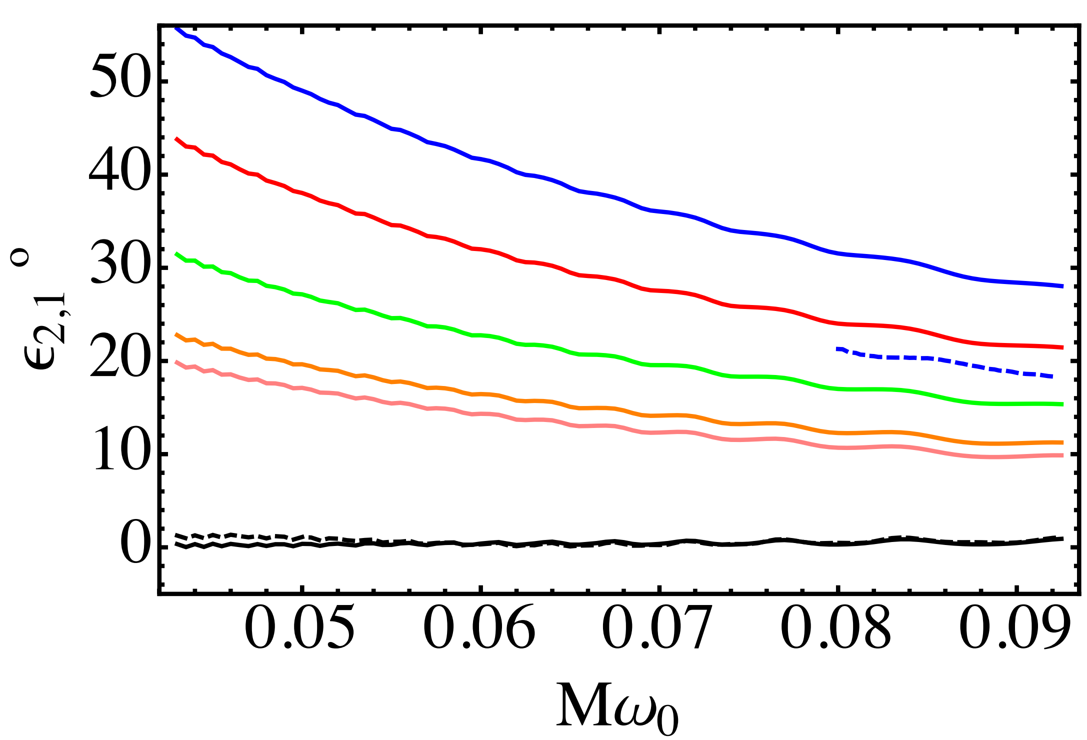
Residual dephasing after alignement at the center of the matching window

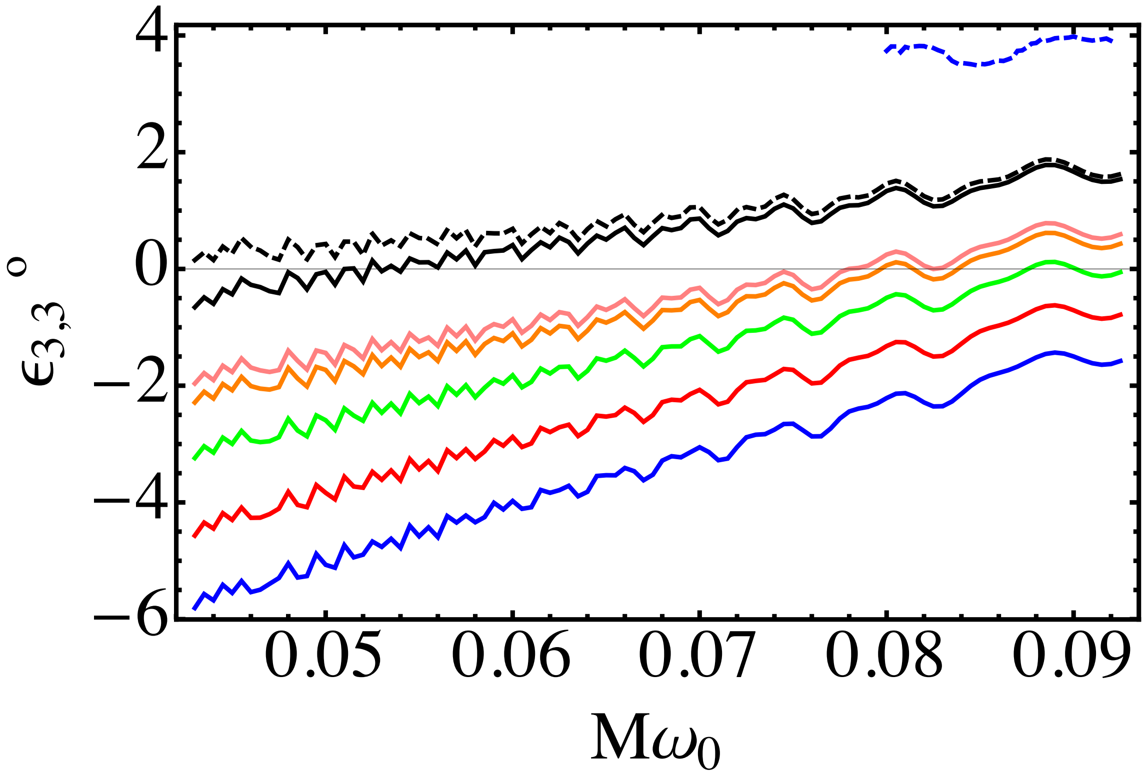
$$\epsilon_{\ell m} = \Delta\phi_{\ell m} + \psi_0 + m\varphi_0,$$

Up to redefinition of the polarization, this is: $\epsilon_{\ell,m}(\omega_0) = \left(\phi^{\rm NR}_{\ell m} - \frac{m}{2}\phi^{\rm NR}_{22}\right) - \left(\phi^{\rm PN}_{\ell m} - \frac{m}{2}\phi^{\rm PN}_{22}\right)$

During the inspiral, in PN, $\phi_{\ell m} \simeq m\phi_{\rm orbital}$

Vary the extraction radius of the waves





Even at very large finite extraction radius, large disagreement for (l,l-1) modes.

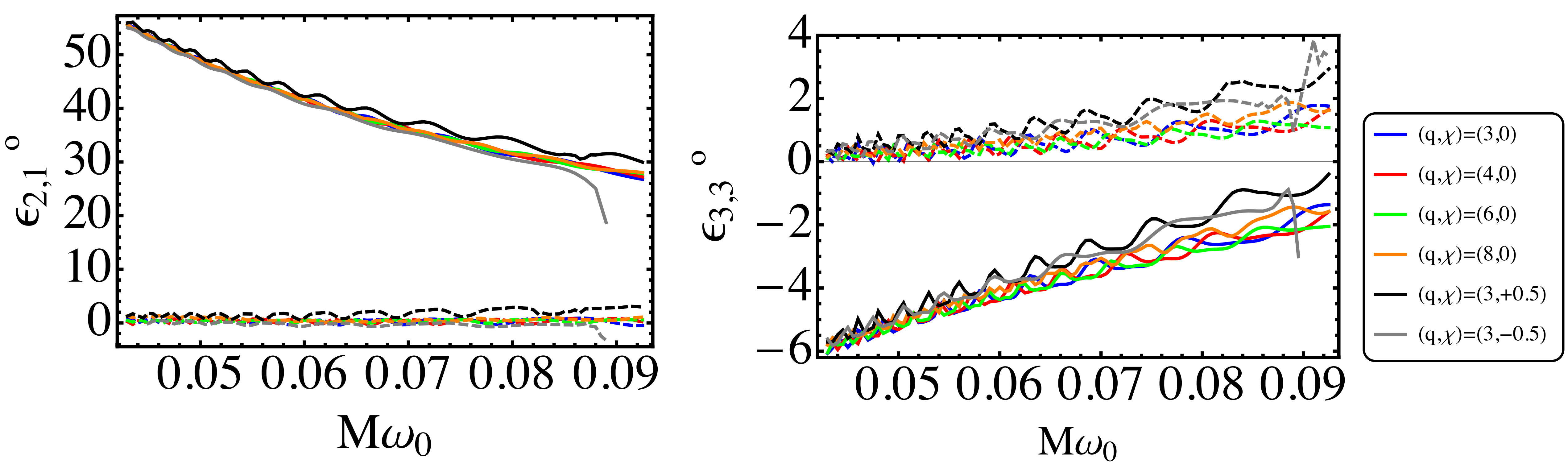
Origin of the phase discrepancies

Slow convergence with extraction radius

fit to $1^\circ (r/r_0)^n$

(ℓ, m)	$(2,1)$	$(3,2)$	$(3,3)$	$(4,3)$	$(4,4)$
n	-0.967	-1.015	-0.941	-1.038	-0.947
r_0	3199	4215	293	4182	598

To a good approximation independent of the physical configuration. Really a property of the code.



Impact of extraction radius on DA

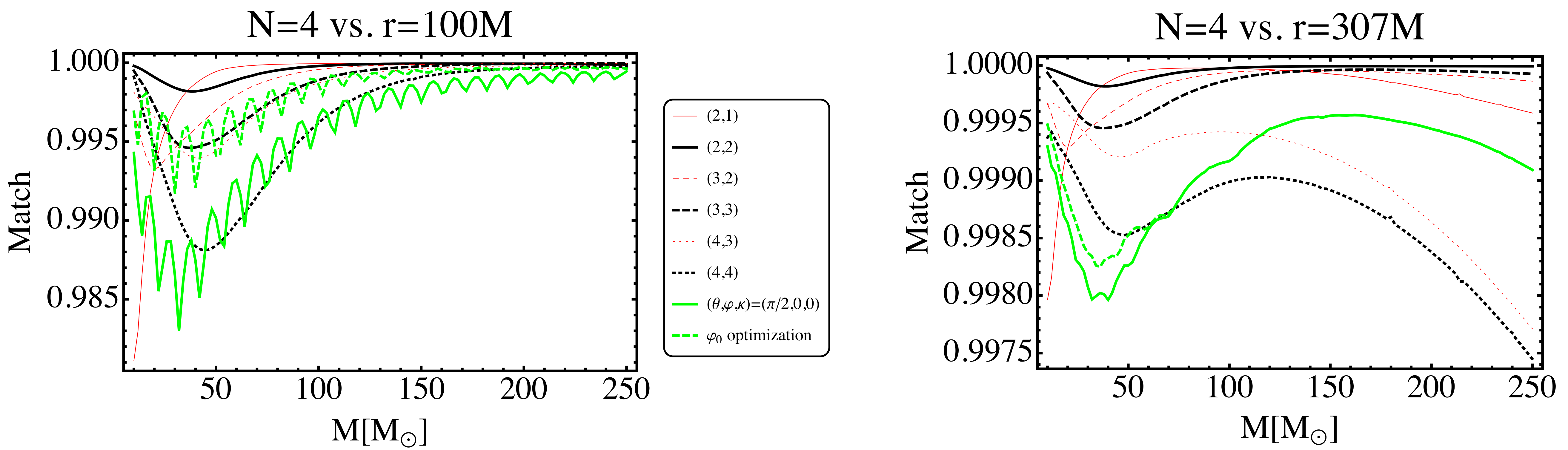
Inner product between waveforms $\langle h_1 \mid h_2 \rangle = 4\Re \int_0^\infty \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df$

Normalized overlap $\mathcal{O}[h_1, h_2] \equiv \frac{\langle h_1 \mid h_2 \rangle}{\sqrt{\langle h_1 \mid h_1 \rangle \langle h_2 \mid h_2 \rangle}}$

+ optimize over time shifts and phase shifts in the (2,2) mode case

$$\max_{t_0} \mathcal{O}\left[h_1(\pi/2, 0, 0), h_2(\pi/2, 0, 0)\right],$$

$$\max_{t_0, \varphi} \mathcal{O}\left[h_1(\pi/2, 0, 0), h_2(\pi/2, \varphi, 0)\right]$$



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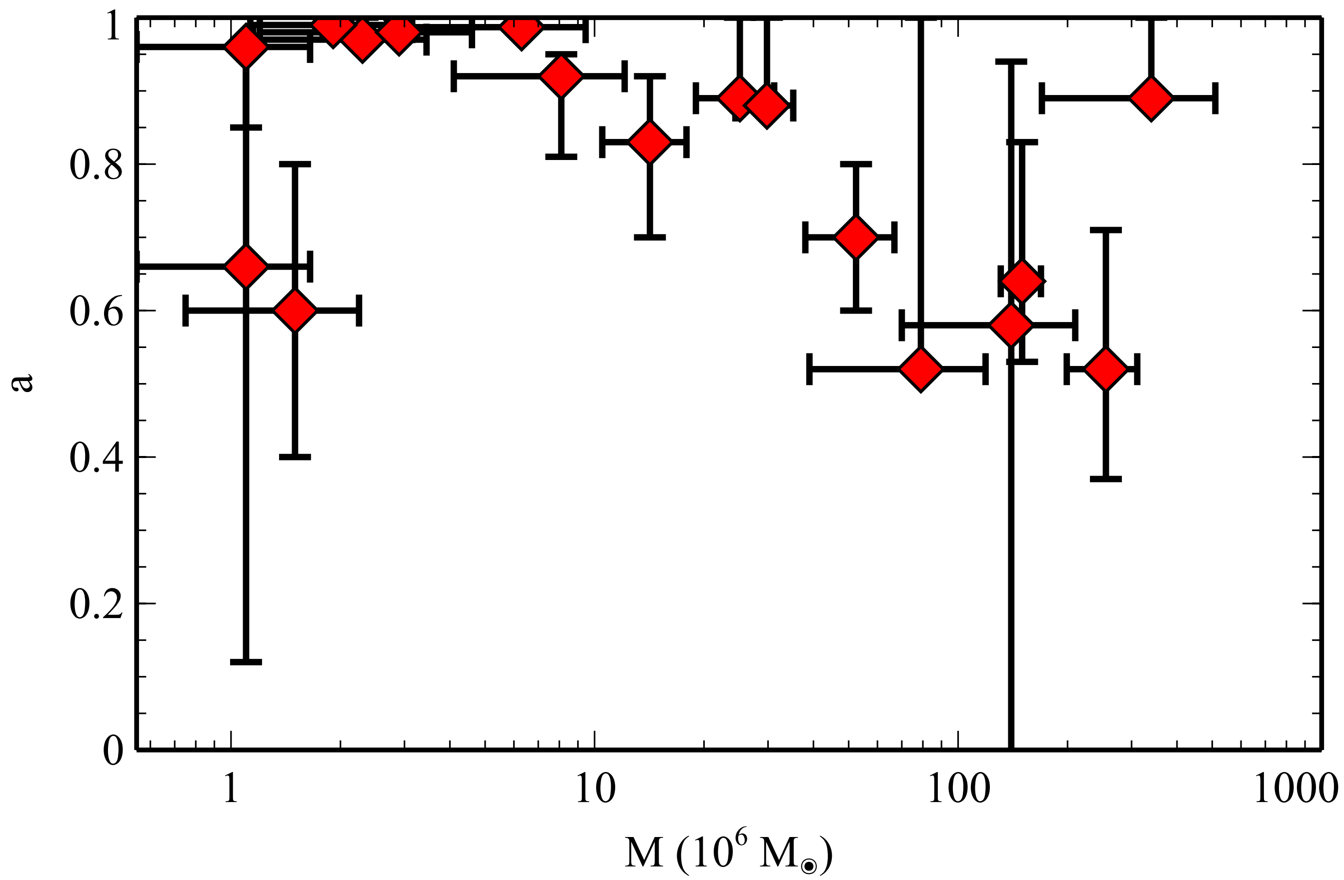
Spinning black holes and neutron stars

Recent astrophysical observations indicate that black holes generically have (large!) spins



taken from Reynolds astro-ph.HE 1302.3260 (2013)

Supermassive Black Holes

similar picture for
Stellar Mass Black holes

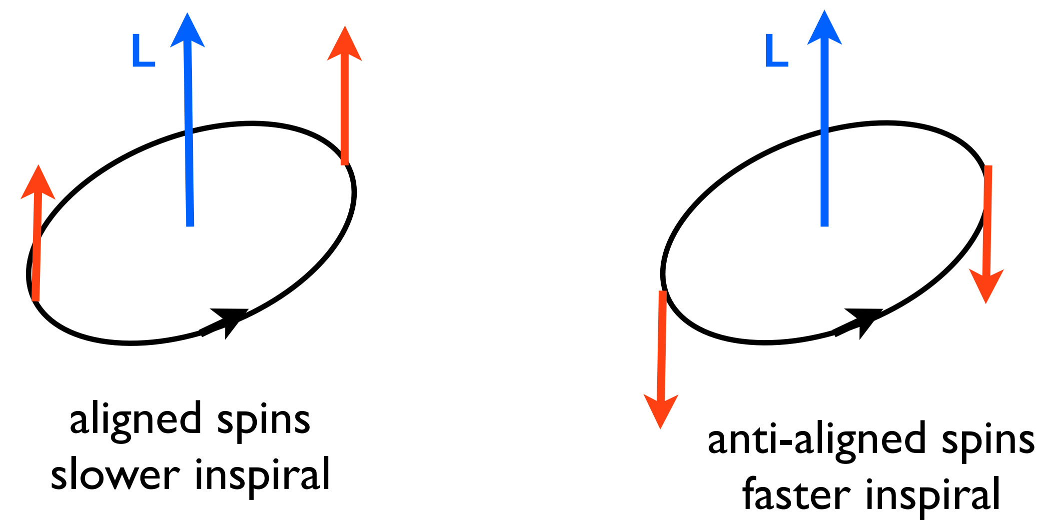
For Neutron Stars: largest dimensionless spin observed $\chi \sim .4$

(in a binary but companion not a compact object)

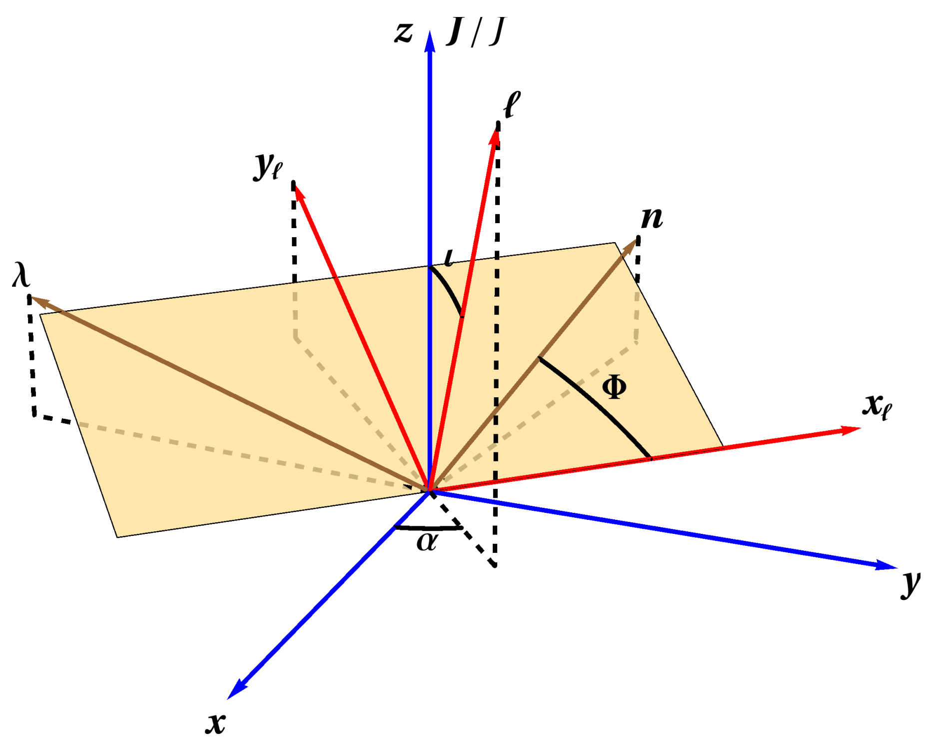
For NS-BH, expected to be lower, by ~ one order of magnitude.

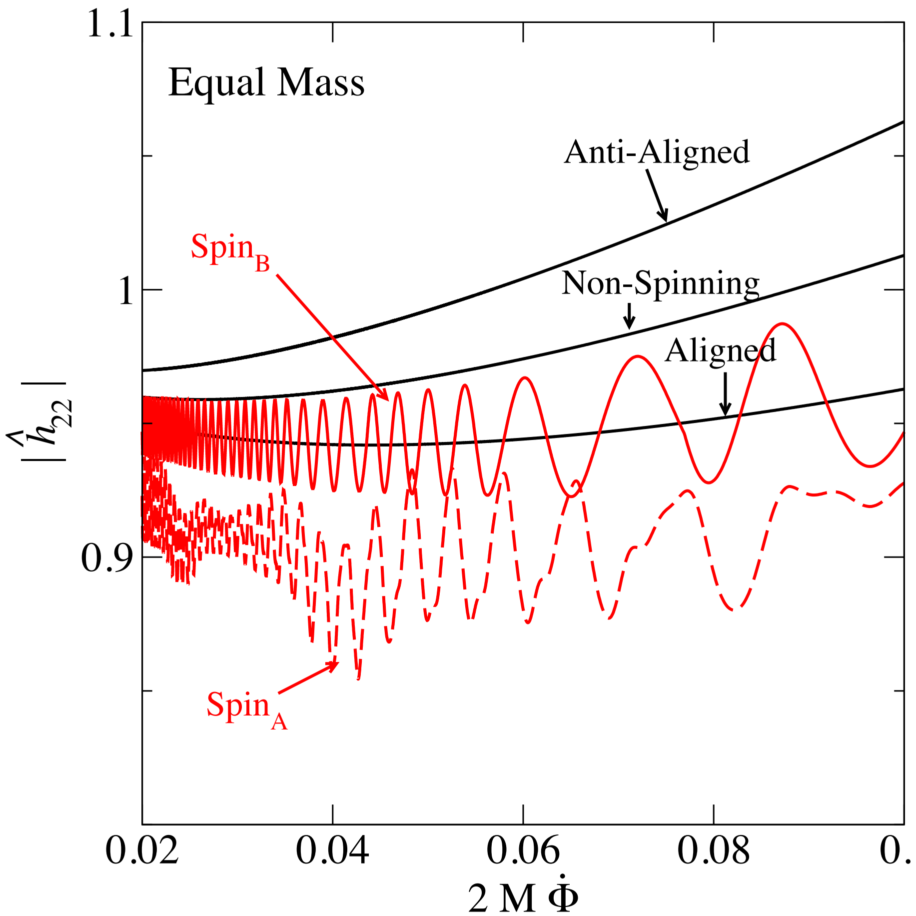
Effect of the spin on the inspiral

The components of the spins that are orthogonal to the orbital plane change the inspiral rate, i.e. in particular the phase



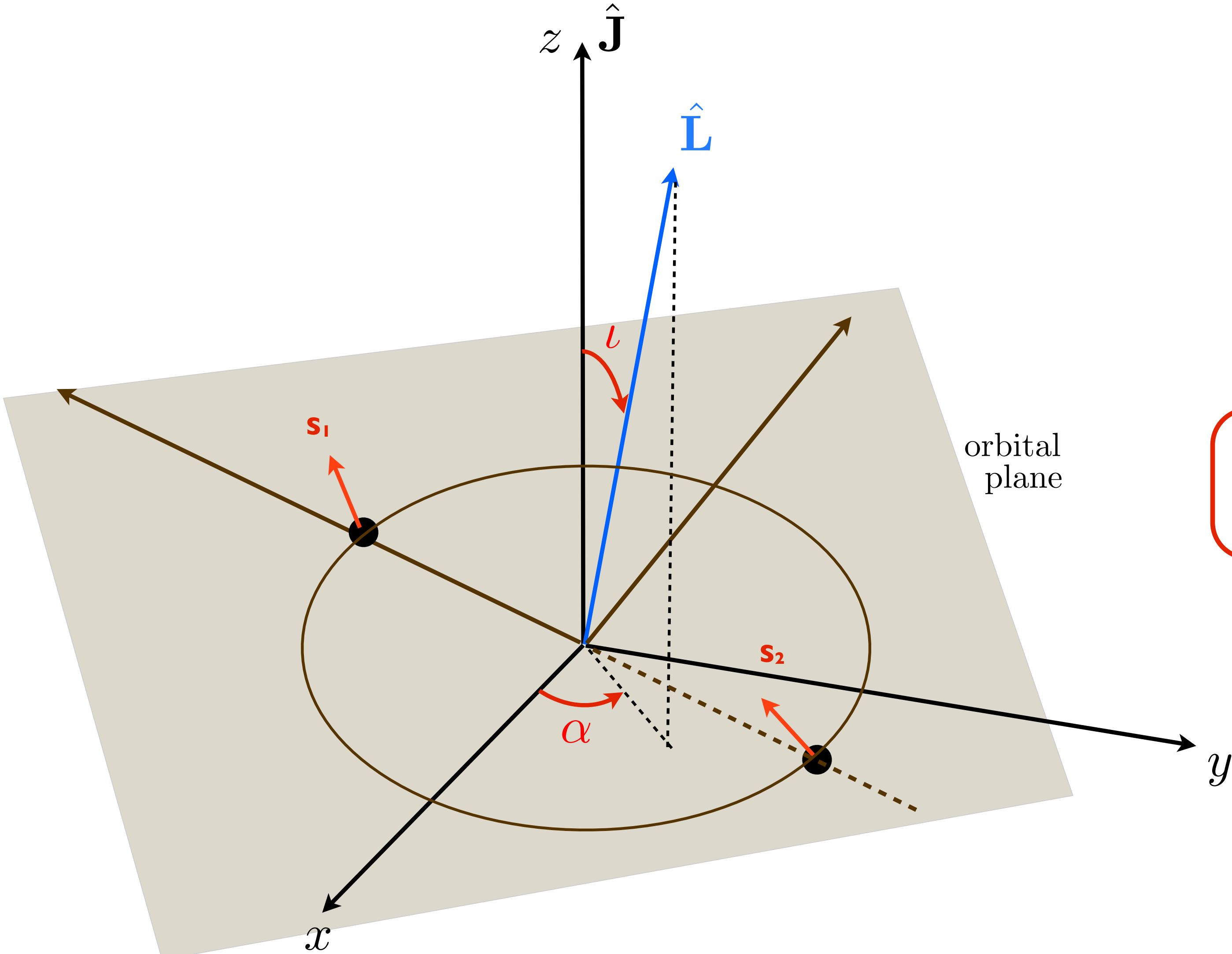
The components of the spins in the orbital plane cause the orbital plane to precess: strong amplitude modulations





taken from Arun, Buonanno, Faye, Ochsner (09)

Dynamics of precession



$$\mathbf{J} = \mathbf{L} + \mathbf{S_1} + \mathbf{S_2}$$

L orbital angular momentum

3 timescales: $t_{\rm orb} \ll t_{\rm prec} \ll t_{\rm rad.reac.}$

On the orbital timescale: **J**, **L**, **S**$_1$, **S**$_2$ fixed

On the precessional timescale: **L**, **S**$_1$, **S**$_2$ precess around **J** which remains fixed

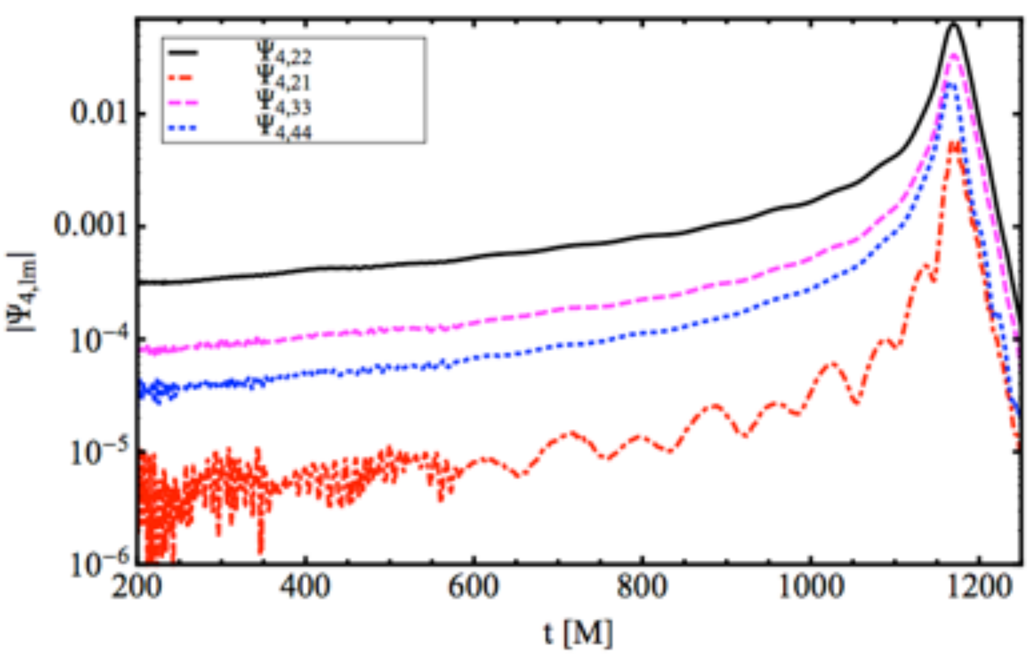
$$\frac{\mathrm{d}\mathbf{S}_1}{\mathrm{d}t} = \mathbf{\Omega}_1 \times \mathbf{S}_1 \qquad \Omega_1 = \frac{1}{c^2}\Omega_1^{\rm 1PN} + \frac{1}{c^4}\Omega_1^{\rm 2PN} + \frac{1}{c^6}\Omega_1^{\rm 3PN} + \mathcal{O}\left(\frac{1}{c^7}\right) \longrightarrow \dot{\alpha}(t)$$

On the radiation reaction timescale: **J** and **L** shrink but in most cases the orientation of **J** remains constant. ι varies

Factorizing precession effects

Idea: one can factorize the effect of precession by going to a non inertial frame in co-rotation with the system. «Quadrupole alignment»

Precessing waveform + appropriate rotation R(t) ≃ Non Precessing waveform



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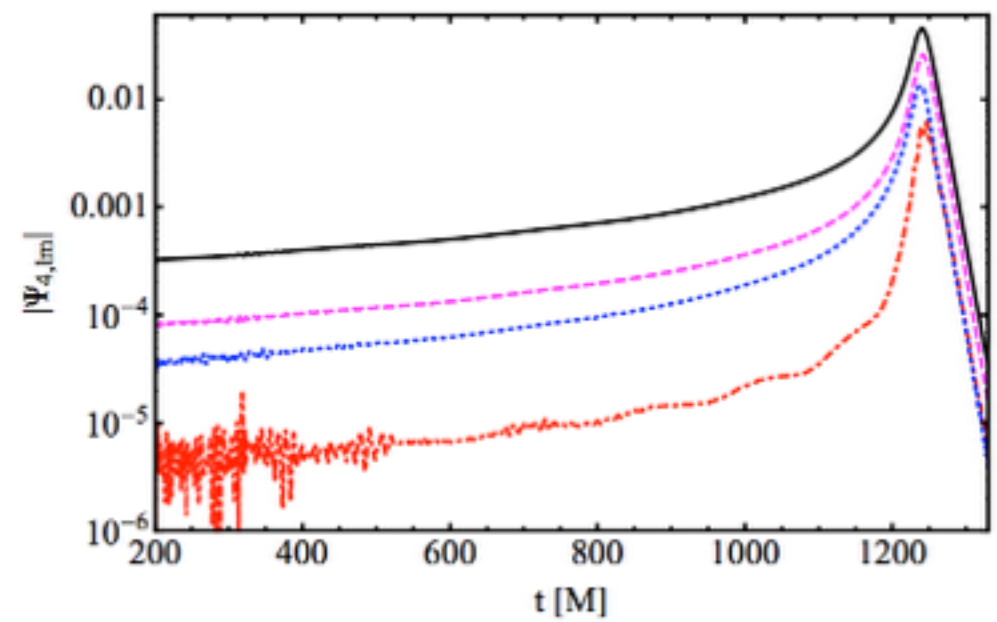


Schmidt et al. (2011, 2013), O'Shaughnessy et al. (2011, 2013), Boyle et al. (2011, 2013), Pekowsky et al. (2013)

The appropriate rotation can be read off the precessing waveform by following the direction that instantaneously maximizes the radiated power.

This closely follows the orbital angular momentum **L**.

⟶ One can model a priori the rotation by solving the precessional dynamics (ι, α)

Twisting up non precessing waveforms

One cheap(er) way of modeling precessing wfs is to model the evolution of **L** i.e. of (ι, α)

- deduce R(t) from EOB dynamics → EOB
- analytical PN prescription → PhenomP

and then twist up a non precessing waveform

$$\dot{\epsilon} = \dot{\alpha} \cos \iota$$

$$h^P_{2m}(t) = e^{-im\alpha} \sum_{|m'|=2} e^{im'\epsilon} d^2_{m',m}(-\iota) h_{2,m'}(t),$$

precessing modes

angle dependent factors

non precessing waveform modes (PhenomC)

- PN angles with NNLO spin-orbit corrections, continued through merger
  see also Ossokine et al. (14), comparisons of the dynamics. Gauge issue…
- model formulated in the frequency domain (faster DA) using the SPA (even through merger!)
- Uses approximate degeneracies 6 → 2 spin params
- Note that no NR precessing simulation was used to formulate the model

Hannam et al., **PRL.**

Inspiral-Merger-Ringdown models for aligned spins

For data analysis purposes, we need models that cover the full coalescence and that are fast to evaluate (purely analytical or solving ODEs)

Two main strategies have been proposed and implemented so far

- Effective One Body formalism (first introduced in Damour, Buonanno (98))
  resummation of the PN results
  map the two body problem to the motion of a test particle in a deformed Kerr metric
  factorized waveform
  calibration to NR
  connection to ringdown: sum of quasinormal modes

- Phenomenological models
  frequency domain
  PN at low frequencies (SPA treatment)
  ansatz fitted to NR simulations for the merger
  effective spin parameter
  connection to ringdown

→ Phenom B/C models for aligned spins

Ajith+ CQG 2007, Ajith+ PRD 2008
Ajith+ PRL 2011, Santamaria+ PRD 2010

aligned IMR Phenom: effective spin

In principle 3 intrinsic parameters: $\eta = \dfrac{m_1 m_2}{(m_1+m_2)^2}, \chi_1 = \dfrac{S_1}{m_1^2}, \chi_2 = \dfrac{S_2}{m_2^2}$

Idea: capture the main features of aligned spin waveforms with as little new parameters as possible (the more params there are, the more expensive the DA). On the other hand, prevents from measuring individual spins...

Fourier domain PN phase:

$$\Psi(f) = \frac{3}{128\eta v^5}\left\{1 + v^2\left[\frac{3715}{756} + \frac{55\eta}{9}\right] - v^3\left[16\pi - \left(\frac{113}{3} - \frac{76\eta}{3}\right)\chi_s - \frac{113\delta}{3}\chi_a\right]\right\} + \dots$$

leading order effect of spin

$$\chi_s = (\chi_1 + \chi_2)/2$$
$$\chi_a = (\chi_1 - \chi_2)/2$$

The effective parameter $\chi \equiv \chi_s + \delta\chi_a - \dfrac{76\eta}{113}\chi_s$

is sufficient to reproduce the leading order effect of spin in the phase. One can rewrite the higher orders in terms of it plus a correction that is ignored.

In fact, for historical reasons, slightly different choice...

(cf Pürrer et al (2013))

IMR Phenom models: aligned spin

$$\tilde{h}_{\rm phen}(f) = A_{\rm phen}(f)\, e^{i\Phi_{\rm phen}(f)}$$

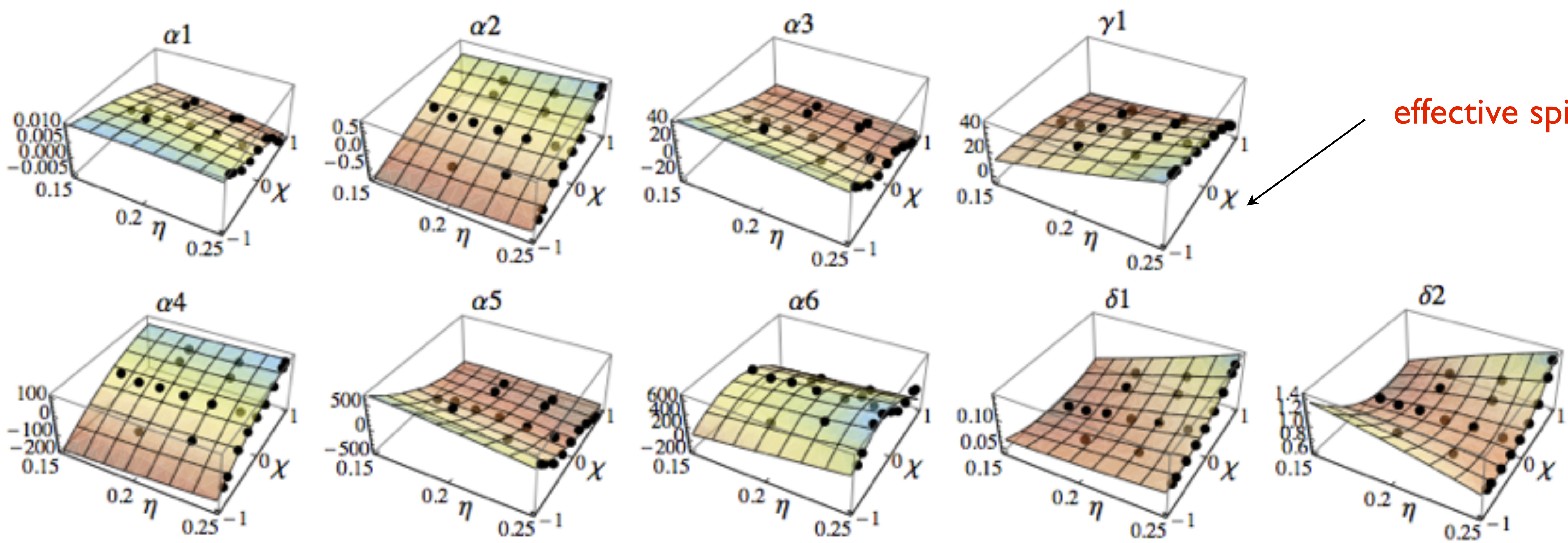
$$\Phi_{\rm phen}(f) = \psi^{22}_{\rm SPA} w^-_{f_1} + \psi^{22}_{\rm PM} w^+_{f_1} w^-_{f_2} + \psi^{22}_{\rm RD} w^+_{f_2}$$

PN RD

$$\psi^{22}_{\rm PM}(f) = \frac{1}{\eta}\left(\alpha_1 f^{-5/3} + \alpha_2 f^{-1} + \alpha_3 f^{-1/3} + \alpha_4 + \alpha_5 f^{2/3} + \alpha_6 f\right)$$

$$w^{\pm}_{f_0} = \frac{1}{2}\left[1 \pm \tanh\left(\frac{4(f-f_0)}{d}\right)\right]$$

Fit of the dependence of the phenomenological parameters on the physical parameters via hybrid waveforms



effective spin

Ajith+ CQG 2007, Ajith+ PRD 2008
Ajith+ PRL 2011, Santamaria+ PRD 2010

Effective precessing spin

Here again, the idea is to minimize the number of «extra» parameters with respect to non precessing models, i.e. to capture the main features of precessing wfs with as little new parameters as possible.

The quantity that affects the phase the most is the precessional speed $\dot{\alpha}$. Its leading order in PN can again be described by some combination of the spins, but it is not constant!

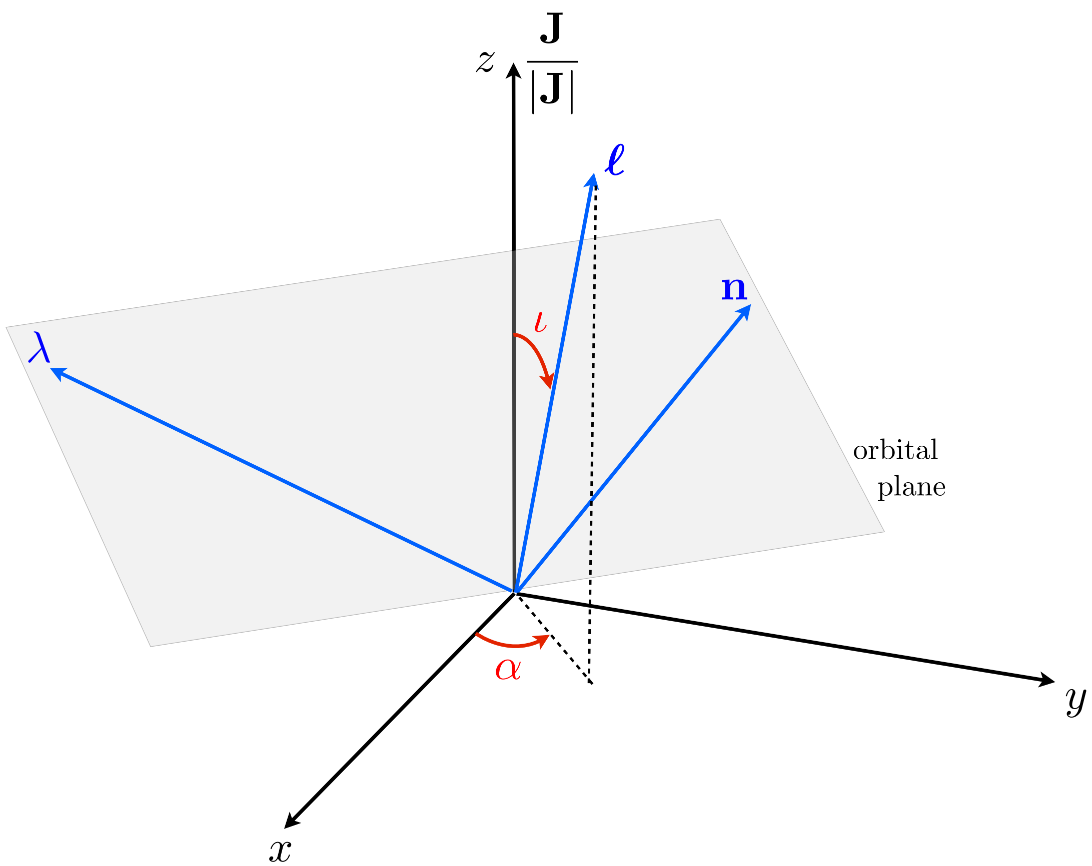
We use the following strategy to restrict ourselves to ONE extra spin parameter:

- consider a single spin system
- average the PN precessional equations over the orientation of the spin in the orbital plane
- the averaged equations now only depend on χ_p and the effective aligned spin $\chi_{\rm eff}$

Our new parameter has a simple interpretation in the single spin case. In the double spin case, we expect that some value will allow to capture the main effects. (presumably the one that reproduces the averaged LO of $\dot{\alpha}$)

Note that from the point of view of data analysis, this doesn't just mean one extra parameter: source orientation and polarization now have to be taken into account!

PN description of the precessional angles



see Blanchet, Faye, Buonanno (06)
Marsat, Bohe, Blanchet, Buonanno (14)

Neglect radiation reaction: $\mathbf{J}$ conserved

Expression of $\mathbf{J}$ in terms of the spin components is known to 3.5PN (NNLO) at the spin orbit level.

$$\cos\iota = \boldsymbol{\ell}\cdot\frac{\mathbf{J}}{|\mathbf{J}|} = \frac{J_l}{\sqrt{J_\ell^2 + J_n^2 + J_\lambda^2}}$$

$$\frac{d\alpha}{dt} = -\frac{\omega_{\rm prec}}{\sin\iota}\frac{J_n}{\sqrt{J_n^2 + J_\lambda^2}}$$

Reduce to 2 effective spin parameters:

single spin + orbital average to reexpress the rhs in terms of the conserved at SO level magnitude of the in plane spin

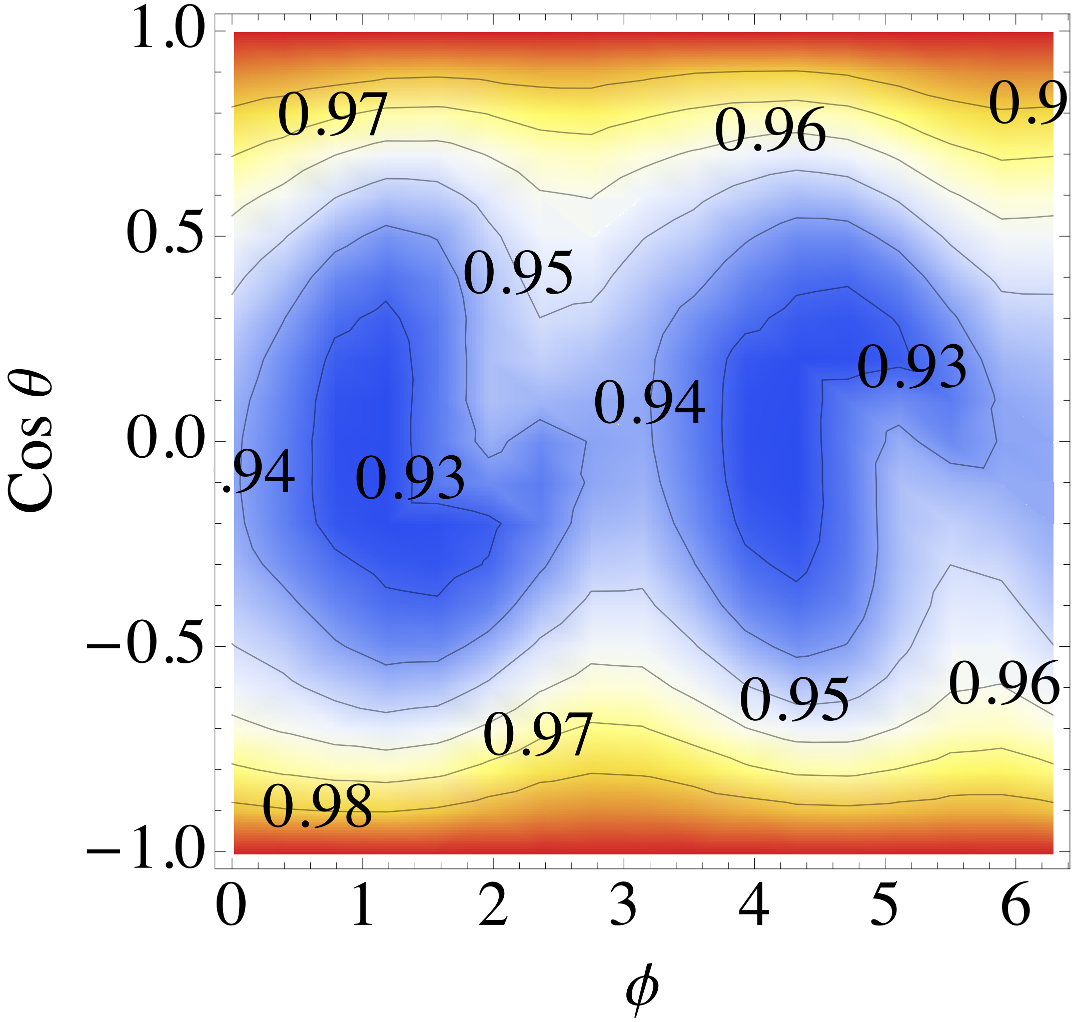
$$+ \qquad \frac{d\alpha}{d\omega} = \frac{1}{\dot\omega}\frac{d\alpha}{dt} \longrightarrow \alpha(\omega)$$

(bring back rad. reaction)

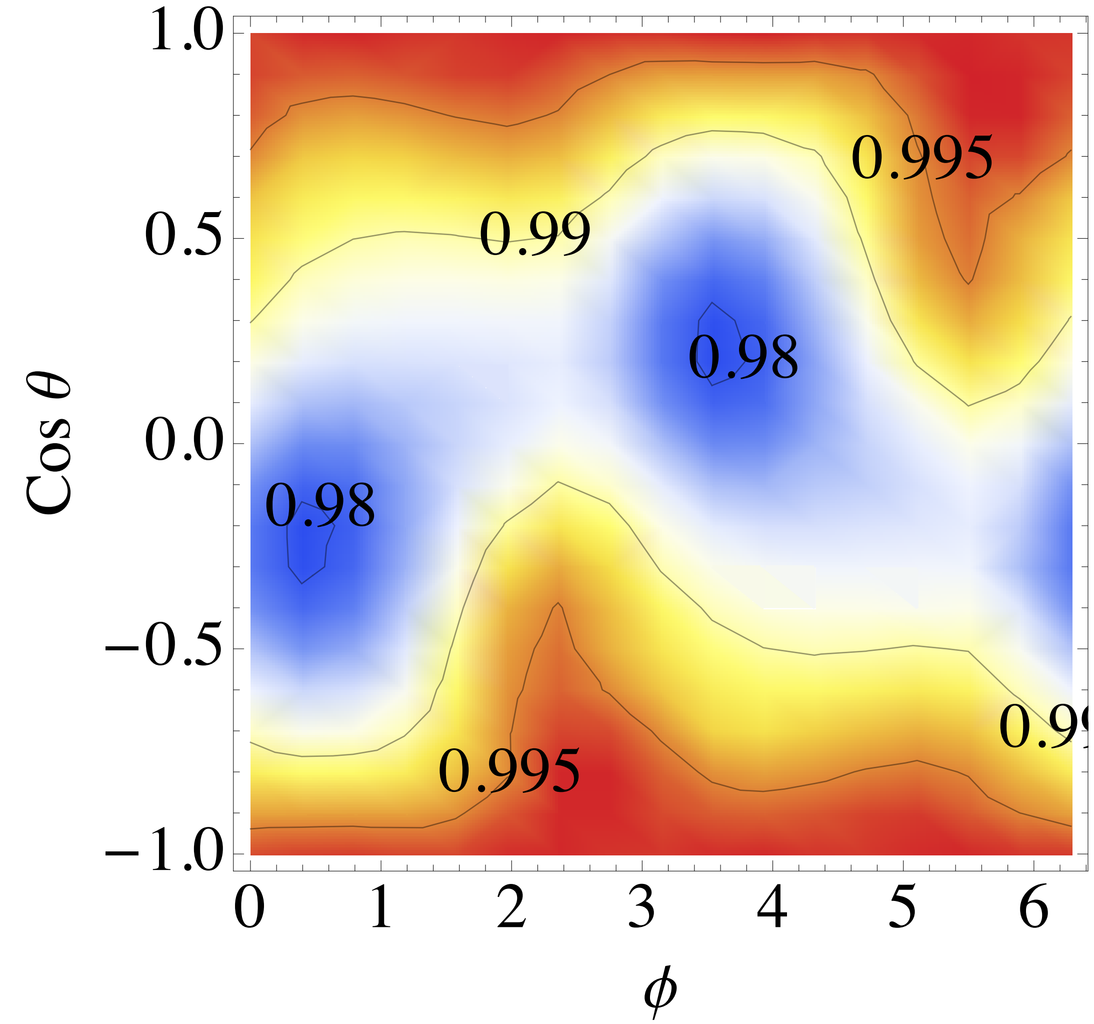
Closed form expressions for the angles in the frequency domain! Does it behave through merger?

PhenomP: effectualness study

PhenomC



PhenomP

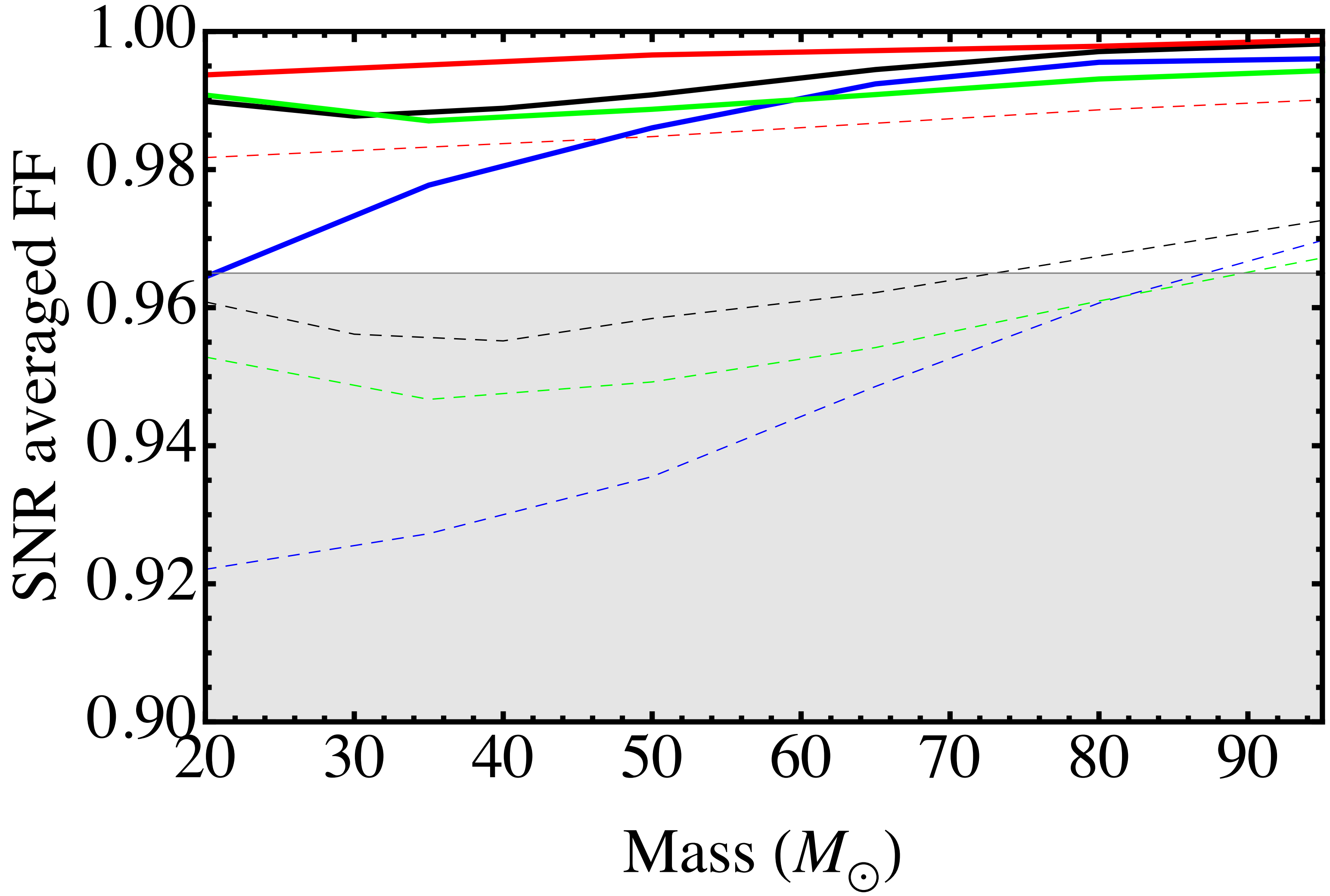


Fitting Factors against a PN-NR hybrid waveform with
50M, fixed polarization, q=3, single spin 0.75 in the plane

$$\langle h_1 | h_2 \rangle = 4\,\mathrm{Re} \int_{f_{\min}}^{f_{\max}} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df$$

Fitting factor = overlap optimized
over the whole freedom in the model

PhenomP: FF for various physical configurations



$q = 3,\ \chi_{\rm eff} = 0$, double spin in the orbital plane

$q = 3,\ \chi_{\rm eff} = 0,\ \chi_p = 0.75$

$q = 3,\ \chi_{\rm eff} = -0.125,\ \chi_p = 0.75$

$q = 3,\ \chi_{\rm eff} = -0.5,\ \chi_p = 0.6$

The model has very high fitting factor to PN/NR hybrids

Next steps

- Refining the model:
  - Easy to "update" as the underlying aligned spin model is refined (PhenomD, calibrated to more NR waveforms coming soon).
    (in collaboration with Husa (UIB), Hannam, Pürrer (Cardiff))
  - Also calibrate the rotation during the merger ringdown.
- First IMR model fast enough to be usable in data-analysis
  - Study the possibility of doing a precessing search in Advanced LIGO (so far, only aligned spin search, and for the first time)
    (in collaboration with Buonanno, Harry, Privitera (AEI, Potsdam))
  - Parameter estimation studies: can we tell if a system is precessing?
    (in collaboration with Hannam, Pürrer (Cardiff), Vitale (MIT))

Conclusions

The perturbative post-Newtonian approach to the coalescence of compact binaries and the numerical description of the merger can be combined in several fruitful ways to produce accurate inspiral-merger-ringdown waveforms.

In this talk, I have discussed,

- construction of hybrid waveforms with higher modes
- PN description of the precessional dynamics as an ingredient of a full IMR analytical model

Many other possible fruitful interactions: calibrate (ingredients of) IMR models, discriminate between inspiral only Taylor approximants, identify "efficient" gauge choices for NR…